A generalization of the SBAS approach to estimate the temporal evolution of Precipitable Water Vapour from time-series of InSAR interferograms


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Outline

• Precipitable Water Vapor (PWV) and InSAR
• PWV temporal evolution by InSAR
• Short-temporal baseline approach for PWV
• B-splines and Continuous-time modelling of PWV
• Results
Precipitable Water Vapour and InSAR

Let us suppose to have an interferogram corrected for topography

\[ \phi(t_M, t_S) = \left[ \psi_{\text{def}}(t_S) - \psi_{\text{def}}(t_M) \right] + \left[ \psi_{\text{atm}}(t_S) - \psi_{\text{atm}}(t_M) \right] \]

where

\[ \psi_{\text{def}}(t) = \frac{4\pi}{\lambda} d(t) \]

\[ \psi_{\text{atm}}(t) = \frac{4\pi}{\lambda} \frac{1}{\Pi \cdot M(\mathcal{G})} PWV(t) \]

\[ M(\mathcal{G}) = \frac{1}{\cos \mathcal{G}} \]

is the mapping function

\[ \Pi = \frac{10^6}{\rho_{H_2O} \cdot R_v \cdot \left[ \frac{k_3}{T_m} + k_2' \right]} \]
Precipitable Water Vapour and InSAR

If terrain deformation can be neglected, InSAR can provide maps of PWV temporal changes

\[ \Delta PWV(t_M, t_S) = \Pi \cdot \frac{\lambda}{4\pi} \cdot \Delta \phi \cdot M_r \]

A set of independent measurements of \(\Delta PWV\) made by a network of permanent GPS stations can be used to calibrate InSAR measurements. Each station measures the mean PWV in a circular area with a radius of about 3.8 km depending on the cut-off angle set in the GPS processing. The idea is to use GPS estimates of PWV at the acquisition times of master and slave SAR images to compute an independent set of \(\Delta PWV\).

Mateus et al., “Can spaceborne SAR interferometry be used to study the temporal evolution of PWV?”, Atmospheric Research, 119, 70-80, 2013
Precipitable Water Vapour and InSAR
Precipitable Water Vapour and InSAR

30/08/2009 → 04/10/2009

Latitude [deg] Longitude [deg] GPS stations

ΔPWV [mm]

MAPR GRIB ALCO FCUL PARC IGPO VNOV PALM ARRA

INSTITUTO DOM LUIZ (IDL)
FRINGE 2015 WORKSHOP
Advances in the Science and Applications of SAR interferometry and Sentinel-1 InSAR Workshop
Precipitable Water Vapour and InSAR

![Map and graph showing Precipitable Water Vapour and InSAR](image)

- **Latitude [deg]**
- **Longitude [deg]**
- **GPS stations**

**04/10/2009 → 08/11/2009**

**Graph:**
- InSAR (black line)
- GPS (red line)

**Precipitable Water Vapour (PWV) in mm**

**GPS stations:**
- MAFR
- GRIB
- ALCO
- FCUL
- PARC
- IGP0
- VNOV
- PALM
- ARRA
Precipitable Water Vapour and InSAR

12/04/2009 → 17/05/2009

GPS stations

MAFR GRIB ALCO FCUL PARC IGP0 VNOV PALM ARRA

Longitude [deg]

Latitude [deg]

ΔPWV [mm]

[mm]
Precipitable Water Vapour and InSAR

![Map and graph showing Precipitable Water Vapour changes between 21/06/2009 and 26/07/2009, with GPS stations marked.](image)
Precipitable Water Vapour and InSAR

26/07/2009 → 30/08/2009

Longitude [deg]

Latitude [deg]

ΔPWV [mm]

GPS stations

MAFR GRIB ALCO FCUL PARC IGP0 VNOV PALM ARRA

[Map and graph showing changes in precipitable water vapour over time and GPS stations]
A refinement of the $\Delta$PWV can be obtained by accurately estimating the mean vertical temperature used to compute the constant $\Pi$.

Usually $T_m$ is obtained by a linear regression with the surface temperature $T_s$.

$$T_m = 47.2 + 0.8T_s$$

rms = 2.59 K
A refinement of the ΔPWV can be obtained by properly estimating the mean vertical temperature used to compute the constant Π.

Usually Tm is obtained by a linear regression with the surface temperature Ts

\[
\xi = 10^{-6} R_v \cdot \left[ \frac{k_3}{T_m} + k'_2 \right]
\]

How to get estimates of absolute PWV?

Relative PWV
InSAR
17/05/09 – 12/04/09

Absolute PWV
WRF model
12/04/09

Absolute PWV
17/05/09
What can we do with InSAR estimates of PWV?

InSAR interpolation to 10km x 10km grid
Short temporal baseline approach

Given N+1 SAR images, sorted by acquisition times, we generate N interferograms by selecting the (master, slave) couples with the shortest temporal baseline.

We denote with $t_s = (t_{s_1}, ..., t_{s_N})$ and $t_m = (t_{m_1}, ..., t_{m_N})$ the acquisition times of master and slave images used to generate the N interferograms ($t_{m_j} > t_{s_j}$).

Considering the j-th interferogram, in every pixel we have

$$
\varphi_j = \frac{4\pi}{\lambda} \cdot \frac{1}{\Pi \cdot M(\theta)} \left[ PWV(0, t_{s_j}) - PWV(0, t_{m_j}) \right] = \psi(t_{s_j}) - \psi(t_{m_j})
$$

The problem consists in estimating the unknown vector $\psi = (\psi(t_1), ..., \psi(t_{N+1}))$ from the vector of known interferogram phase values $\varphi = (\varphi(t_1), ..., \varphi(t_N))$

$$
\min_{\psi \in \mathbb{R}^{N+1}} \| A\psi - \varphi \|^2
$$
Short temporal baseline approach

Supposing that $\psi(t)$ can be expressed as a linear model depending on a vector $p$ of length $L$, the problem can re-stated as finding the unknown vector $p$ from measured values $\varphi = (\varphi(t_1), \ldots, \varphi(t_N))$

$$\min_{\psi \in \mathbb{R}^{N+1}} \|Sp - \varphi\|^2$$

The structure of matrix $S$ depends on how $S$ is related to $\psi(t)$

If $\psi(t)$ has dimensionality $L$, the final problem is given by

$$\min_{\varphi \in \mathbb{R}^{N}} \|\tilde{\psi} - \varphi\|^2$$
Cubic B-spline model

**Definition 3.1** Given a sorted sequence \( \{T_\nu\}_{\nu \in \mathbb{N}} \subset \mathbb{R} \), a B-spline of degree \( n \) is a spline function \( s(t) \) such that

- \( s(t) \) is non-zero only within an interval \([T_\nu, T_{\nu+n+1}]\) for a certain \( \nu \)
- \( \int_{T_\nu}^{T_{\nu+n+1}} s(t) dt = 1 \)
- in any interval \([T_i, T_{i+1}]\) \( i = \nu, \ldots, \nu + n \) is a polynomial of degree at most \( n \)
- \( s(t) \in C^{(n-1)}(\mathbb{R}) \)

Such function \( s(t) \) is usually noted as \( B_{n,\nu}(t) \).

For example a cubic B-spline (\( n=3 \)) has a support \([T_\nu, T_{\nu+n+1}]\) and is a cubic polynomial in any of the four intervals \([T_\nu, T_{\nu+1}], [T_{\nu+1}, T_{\nu+2}], [T_{\nu+2}, T_{\nu+3}], [T_{\nu+3}, T_{\nu+4}]\) composing its support.
Cubic B-spline model

If a n-degree spline function $s(t)$ is defined on an interval $A$ partitioned by a finite sequence $t_0 < t_1 < \ldots t_{M+1}$, it can be expressed as

$$s(t) = \sum_{\nu=-n}^{m} p_{\nu} \cdot B_{n,\nu}(t)$$

where $t_{-N} < \ldots t_{-1} < t_0 < t_1 < \ldots t_{M+1} < \ldots t_{M+N+1} = t_L$ are called control points.

Thanks to this theorem the approximation by means of n-th degree splines can be achieved by a sum of B-splines provided each interval $[t_i, t_j]$ contains at least an observation point the Schoenberg-Whitney condition.

Useful for studying $\Delta PWV(t)$ characterized by a good spatial correlation but poor temporal correlation.
A set of 11 Envisat-ASAR images acquired over the Lisbon area are used.

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</tbody>
</table>

- Support of $B_{3,-3}$
- Support of $B_{3,4}$

$n = 3$
$m = L - (n+1) = L-4$

T-3 T-2 T-1 T0 T1 T2 T3 T4 T5 T6 T7 T8

28/12/2008 08/11/2009
Cubic B-spline model

In our case we can use B-splines so that we express

\[ PWV(t) = \sum_{\nu=-3}^{L-4} p_{\nu} \cdot B_{3,\nu}(t) \]

The j-th interferogram is then given by

\[ \varphi_j = \varphi(j) - \varphi(j-1) = \frac{4\pi}{\lambda} \cdot \frac{1}{\Pi \cdot M(\vartheta)} \sum_{\nu=-3}^{L-4} p_{\nu} \cdot [B_{3,\nu}(j) - B_{3,\nu}(j-1)] \]

and we can formulate the linear system

\[ S \cdot p = \varphi \]

where S is generally a sparse matrix. Then the problem to be solve is

\[ \min_{\varphi \in \mathbb{R}^L} \left\| \tilde{S} p - \varphi \right\|^2 \]
Spatial block system and physical constraints

We adopt the B-splines model with a number $L = 7$ of B-splines. For a given pixel the least-squares problem in solved in $\mathbb{R}^7$

$$\min_{\psi \in \mathbb{R}^L} \left\| S \, p - \phi \right\|^2$$

The interferograms are partitioned in $3 \times 3$ non-overlapping windows and the least squares problem is jointly solves for the 9 pixels of each window. The dimensionality of the problem increases to $7 \times 9 = 63$.

$$-b_i \leq p_p(l) - p_{Q_p}(l) \leq -b_i \quad \text{with} \quad l = 1, \ldots, L$$
Spatial block system and physical constraints

Linear constraints are added by imposing bounds on element of $p$ relating adjacent pixels following the scheme (a total number of $7 \times 8 = 56$ box constraints are obtained for each problem)

$$-b_l \leq p_p(l) - p_{Qp}(l) \leq b_l \quad \text{with} \quad l = 1, \ldots, L$$
Spatial block system and physical constraints

\[
\begin{align*}
\min_{\mathbb{R}^{63}} & \left\| \begin{pmatrix} S & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 \\ \vdots \\ 0 & & & S \\ \end{pmatrix} \begin{pmatrix} p_1 \\ \vdots \\ p_9 \\ \end{pmatrix} - \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_9 \\ \end{pmatrix} \right\|^2 \\
\text{subject to} & \quad (b > 0) \\
& \quad -b \leq p_p(1) - p_{q_p}(1) \leq b \\
& \quad \vdots \\
& \quad -b \leq p_p(7) - p_{q_p}(7) \leq b \\
& \quad \text{For each pair of adjacent pixels} \\
\end{align*}
\]

The same value of $b > 0$ is used for all constraints

The aim is to find a good continuous-time model for $\text{PWV}(t)$
Results (b = 20)
Results (b = 20)
Results (b = 0.1)
Results (b = 0.1)
Conclusions and future work

• The original SBAS method has been adopted to generate maps of PWV and extended by adding spatial constraints to help in finding a suitable solution

• The problem modelling temporal evolution of PWV maps generated by InSAR has been studied by means of B-splines

• The future work will focus on extending the proposed method to include GPS-derived PWV time series
Results (b = 10)
Results (b =10)
Results ($b = 5$)
Results (b = 5)
Results (b = 3)
Results ($b = 3$)
Results ($b = 1$)
Results (b = 1)
Results ($b = 1$)
Results ($b = 1$)