Advanced InSAR processing in the footsteps of SqueeSAR

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SqueeSAR


SqueeSAR can be considered as spatio-temporal filtering of an InSAR data stack that converts DS into PS. It runs through the following steps:

1. Grouping: For every pixel statistically similar pixels are sought in its vicinity. A neighborhood $\Omega$ is formed of those accepted.

2. Phase triangulation (phase linking): The ML-estimator for the interferometric phase introduced by Monti Guarnieri and Tebaldini is applied to the pixels of $\Omega$.

3. In case phase linking is successful, the pixel’s phase is replaced by the estimated phase.

4. PS and converted DS are fed into a PS-algorithm.
Agenda

1. Augmentation of the stochastic model underlying phase linking
2. Estimators of scatter (covariance)
3. Results of tests versus ground truth
4. Summary and outlook
Monti Guarnieri, Tebaldini: Stochastic Model for Phase Linking

DS = statistically homogeneous neighborhood $\Omega$ of pixels, assumed to be independently and identically distributed with circular complex normal distribution

$$z(\omega) = \begin{pmatrix} z(\omega)_1 \\ \vdots \\ z(\omega)_N \end{pmatrix}$$

Pixel $\omega$, acquisitions $1, \ldots, N$

$$z = \begin{pmatrix} z(\omega_1) \\ \vdots \\ z(\omega_K) \end{pmatrix} \sim \mathcal{CN}(0, C \otimes I_K, 0) \quad K=\# \Omega, \: I_K = \text{Identity matrix} \quad c_{nm} \in \mathbb{R}_{\geq 0}$$

$$\zeta(\omega_K)_n = z(\omega_K)_n \cdot \exp(i \cdot \phi_n), \forall \omega_K \in \Omega, \phi_n = \alpha_n + \phi^{defo}_n + \phi^{topo}_n + \cdots$$

$$\zeta = \begin{pmatrix} \zeta(\omega_1) \\ \vdots \\ \zeta(\omega_K) \end{pmatrix} \sim \mathcal{CN}(0, C(\phi) \otimes I_K, 0) \quad \phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix}$$

$$C(\phi)_{nm} = c_{nm} \cdot \exp(i \cdot (\phi_n - \phi_m))$$

Interpretation: for all pixels in $\Omega$ we have the same change of phase because of atmosphere, deformation, DEM error, reflectivity changes etc.
Monti Guarnieri, Tebaldini: Phase Linking

The likelihood function is
\[ p(\zeta|\phi) = \text{const.} \cdot \exp(-\zeta^H (C(\phi) \otimes I_K)^{-1} \zeta) . \]

We have to find \( \phi \), such that \( \zeta^H (C(\phi) \otimes I_K)^{-1} \zeta = \cdots = \zeta^H \cdot (|C|^{-1} \cdot I_\Omega) \cdot \zeta \) is minimized.

Thereby
\[ \zeta = \begin{pmatrix} e^{i\phi_1} \\ \vdots \\ e^{i\phi_N} \end{pmatrix}, \phi_1 = 0, (I_\Omega)_{nm} = \sum_{\omega \in \Omega} \zeta(\omega)_n \zeta(\omega)_m^H = \# \Omega \cdot \hat{S}_{nm}. \]

The sample covariance matrix \( \hat{S} \) is the MLE in the normally distributed case.
Augmentation of stochastic Model

$$z = \begin{pmatrix} z(\omega_1) \\ \vdots \\ z(\omega_K) \end{pmatrix} \sim CN(0, C \otimes I_K, 0) \quad K = \# \Omega, \quad I_K = \text{Identity matrix} \quad c_{nm} \in \mathbb{R}_{\geq 0}$$

$$\zeta(\omega_k)_n = (s(\omega_k) \cdot z(\omega_k)_n) \cdot \exp \left( i \cdot ( \phi_n ) \right), \forall \omega_k \in \Omega$$

$$s_k \in \mathbb{R}_{>0} \quad \text{scaling factor (different pixels have different mean backscatter)}$$
Augmentation of stochastic Model

\[
z = \begin{pmatrix} z(\omega_1) \\
\vdots \\
z(\omega_K) \end{pmatrix} \sim CN(0, C \otimes I_K, 0) \quad K = \#\Omega, \ I_K = \text{Identity matrix} \quad c_{nm} \in \mathbb{R}_{\geq 0}
\]

\[
\zeta(\omega_k)_n = (s(\omega_k) \cdot z(\omega_k)_n) \cdot \exp \left( i \cdot (\psi(\omega_k)_n + \phi_n) \right), \forall \omega_k \in \Omega
\]

\[s_k \in \mathbb{R}_{>0} \quad \text{scaling factor (different pixels have different mean backscatter)}\]

\[\psi(\omega_k)_n \in \mathbb{R} \quad \text{residual fringes (gradient of deformation or topography, ...)}\]
Augmentation of stochastic Model

\[ z = \begin{pmatrix} z(\omega_1) \\ \vdots \\ z(\omega_K) \end{pmatrix} \sim \mathcal{CN}(0, C \otimes I_K, 0) \quad K = \#\Omega, \quad I_K = \text{Identity matrix} \quad c_{nm} \in \mathbb{R}_{\geq 0} \]

\[ \zeta(\omega_k)_n = (s(\omega_k) \cdot z(\omega_k)_n + o_{kn}) \cdot \exp \left( i \cdot (\psi(\omega_k)_n + \phi_n) \right), \forall \omega_k \in \Omega \]

- \( s_k \in \mathbb{R}_{>0} \) scaling factor (different pixels have different mean backscatter)
- \( \psi(\omega_k)_n \in \mathbb{R} \) residual fringes (gradient of deformation or topography, …)
- \( o_{kn} \in \mathbb{C} \) object or outlier (assumed to be zero for most \( k,n \))
Augmentation of stochastic Model

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- \( s_k \in \mathbb{R}_{>0} \) scaling factor (different pixels have different mean backscatter)
- \( \psi(\omega_k)_n \in \mathbb{R} \) residual fringes (gradient of deformation or topography, …)
- \( o_{kn} \in \mathbb{C} \) object or outlier (assumed to be zero for most k,n)

\[ \zeta = \begin{pmatrix} \zeta(\omega_1) \\ \vdots \\ \zeta(\omega_K) \end{pmatrix} \sim \mathcal{CN}(\mu(\phi, o, \psi), C(\phi, s, \psi), 0) \quad \text{where} \quad \mu(\phi, o, \psi) = E[\zeta] \]
Phase linking for the augmented stochastic model

The likelihood function now is
\[ p(\zeta | \phi) = \text{const.} \cdot \exp \left( - (\zeta - \mu(\phi, o, \psi))^H \cdot C(\phi, s, \psi)^{-1} \cdot (\zeta - \mu(\phi, o, \psi)) \right). \]

We have to find \( \phi \), which minimizes
\[ (\zeta - \mu(\phi, o, \psi))^H \cdot C(\phi, s, \psi)^{-1} \cdot (\zeta - \mu(\phi, o, \psi)) = \cdots = \xi^H \cdot (|C|^{-1} \cdot I_\Omega) \cdot \xi \]

Where now
\[ (I_\Omega)_{nm} = \sum_{\omega \in \Omega} s(\omega)^{-2} \cdot (e^{-i \cdot \psi(\omega)_n} \cdot (\zeta(\omega)_n - \mu(\omega)_n)) \cdot (e^{-i \cdot \psi(\omega)_m} \cdot (\zeta(\omega)_m - \mu(\omega)_m))^H \]

So we can do the following:
1. Remove the residual fringes.
2. Estimate \( s(\omega_k) \) and scale the data.
3. Apply a robust covariance estimator \( \hat{C} \) in order to cope with outliers.
Phase Linking for complex elliptically symmetric (CE) distributions

The likelihood function in the case of $z(\omega) \sim CE(0, \Sigma, g)$, $\forall \omega \in \Omega$ is

$$p(\zeta|\phi) = \text{const} \cdot g \left( (\zeta - \mu(\phi, o, \psi))^H \cdot \Sigma(\phi, s, \psi)^{-1} \cdot (\zeta - \mu(\phi, o, \psi)) \right).$$

The density generator $g: \mathbb{R}_{\geq 0} \to \mathbb{R}_{>0}$ satisfies $\int_0^\infty t^{N-1} g(t) dt < \infty$.

The scatter matrix $\Sigma$ is proportional to the covariance matrix: $\lambda \cdot \Sigma = C$, $\lambda \in \mathbb{R}_{>0}$.

Given $g$ is strictly monotonic decreasing phase triangulation can be performed as before. We have to find $\phi$, which minimizes

$$\xi^H \cdot \left( |\hat{\Sigma}|^{-1} \circ \hat{C} \right) \cdot \xi = \lambda \cdot \xi^H \cdot \left( |\hat{C}|^{-1} \circ \hat{C} \right) \cdot \xi = \lambda \cdot \xi^H \cdot \left( |\hat{\Sigma}|^{-1} \circ \hat{\Sigma} \right).$$

$g$ is strictly monotonic decreasing e.g. for $t$-distributions, generalized Gaussian distributions, $K$-distributions, inverse Gaussian compound Gaussian distributions.
M-estimators of scatter for CE distributions

For a weight function $\varphi: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ the M-estimator of scatter $\hat{\Sigma}$ is defined as the solution of the equation

$$\hat{\Sigma} = \frac{1}{\#\Omega} \sum_{\omega \in \Omega} \varphi \left( \zeta(\omega)^H \hat{\Sigma}^{-1} \zeta(\omega) \right) \cdot \zeta(\omega) \zeta(\omega)^H$$

We have:

1. For appropriate $\varphi$ the equation is uniquely solvable by iteration.
2. The ML estimators for CE distributions are M-estimators.
3. Robust estimators iff $t \mapsto t \cdot \varphi(t)$ is continuous and bounded.

Robust estimators are e.g. $\text{MLT}(\nu)$ and $\text{Huber}(q)$:

$$\varphi_{\text{MLT}}(t) = \frac{2N + \nu}{2t + \nu}$$

$$\varphi_{\text{Huber}}(t) = \begin{cases} \frac{1}{b(q)}, & t \leq c(q)^2 \\ \frac{c(q)^2}{t \cdot b(q)}, & t > c(q)^2 \end{cases} \quad c(q) = \frac{1}{2} (\chi^2_{2N})^{-1}(q), \quad b(q) = \chi^2_{2(N+1)}(2c(q)^2) + 2c(q)^2 \frac{1-q}{N}$$
Tests regarding estimators of scatter

Investigated were
1. estimators $\text{SampleCM}$, $\text{MLT}(\nu)$, $\text{Huber}(q)$,
2. scaled and nonscaled,
3. different search window sizes (limiting the possible size of $\Omega$) and significance levels during grouping,
4. for two data stacks.
Testing procedure

Preprocessing of selected pixels:
1. Grouping
2. Estimation of scatter matrix
3. Phase linking

DS are evaluated relative to a reference PS-result. Original phases of DS-pixels are replaced by linked phases:
1. APS at DS-positions is interpolated and removed
2. DS are connected to nearby PS and linear velocity trends and DEM errors are estimated
3. Values at DS-positions are determined via inversion

Evaluation of DS-results versus ground truth
Data stack and ground truth Lüneburg

LOS-velocities according to ground truth (Background image from Google Earth)

- 26 TSX HRS (300MHz) acquisitions
- 199 measurement points from levelling available as ground truth for LOS-velocity (provided by the construction office of the town)
- 66 in an area of strong subsidence were used for tests
- LOS-velocities have been assigned to pixel positions in the master
Data stack and ground truth Lüneburg

- 26 TSX HRS (300MHz) acquisitions
- 199 measurement points from levelling available as ground truth for LOS-velocity (provided by the construction office of the town)
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LOS-velocities according to interpolated ground truth
(Background mean amplitude image)
Data stack and ground truth Greding (Bavaria)

- 23 TSX HRS (300MHz) acquisitions
- LIDAR-DTM of Bavarian land surveying office used as ground truth (height accuracy better 0.2m, positional accuracy better 0.5m)
- DTM was transformed to master raster
- 250 pixels selected for tests

Pixels chosen for tests (Background mean amplitude image)
Tests Greding: # pixels with $\gamma \geq 0.7$ and $|\Delta| \leq 1 m$
Tests Lüneburg: #pixels with $\gamma \geq 0.7$ and $|\Delta| \leq 0.002m/y$ 
or with relative error smaller than 10%.
Summary and outlook

1. Presentation of an augmented stochastic model that clarifies how phase linking can be applied when the distribution of data deviates from the assumption of Gaussian distribution.

2. Discussion of phase linking for complex elliptically symmetric distributions.

3. Test results for several estimators of scatter indicating:
   a. Scaling of amplitudes improves results for most estimators of scatter for the stack of Lüneburg.
   b. Robust estimators MLT(ν) and Huber(q) outperform the sample coherence matrix when suitably parameterized.

Thank you for your attention!