Inverse Modelling of Atmospheric Carbon Dioxide

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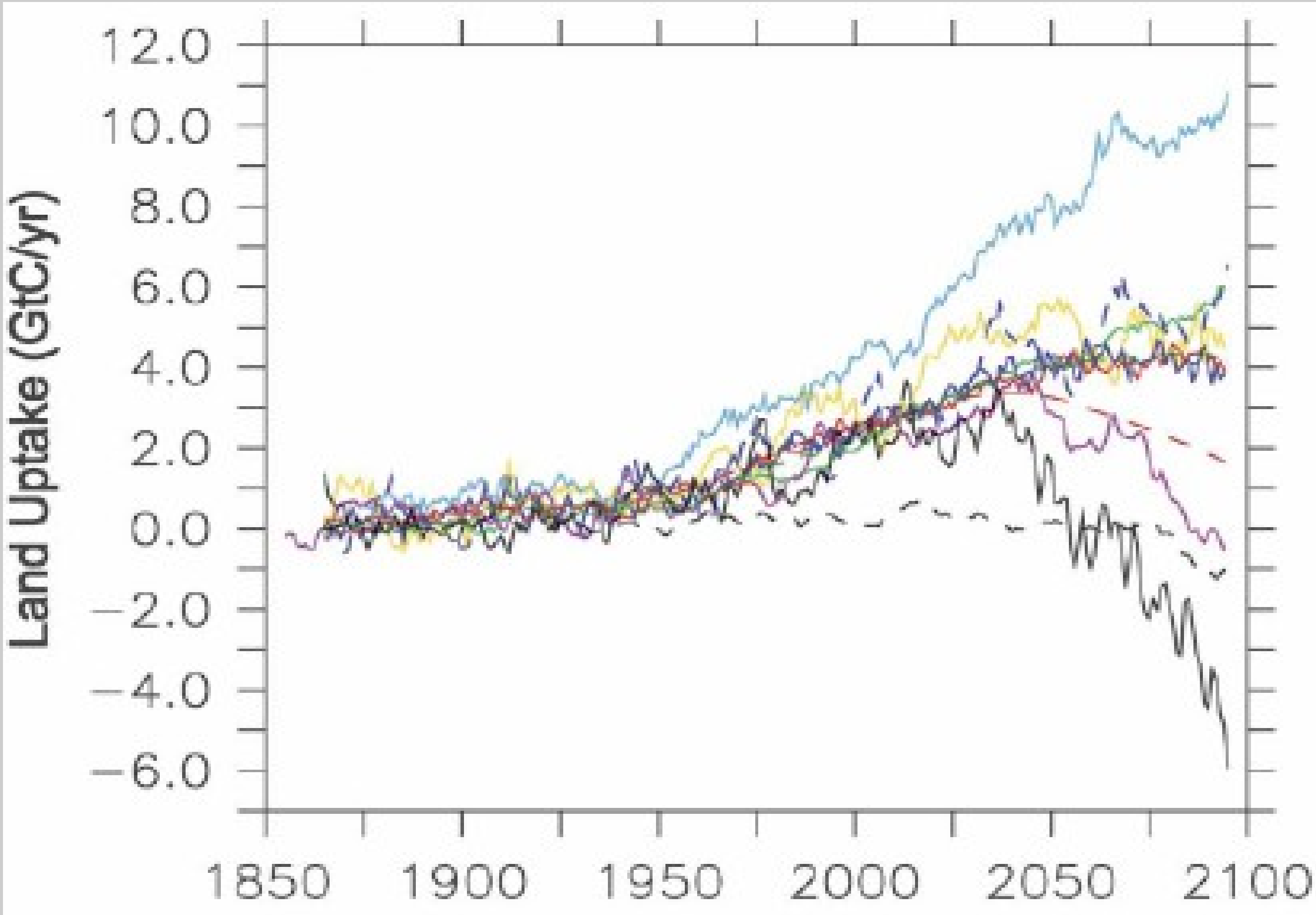
Outline

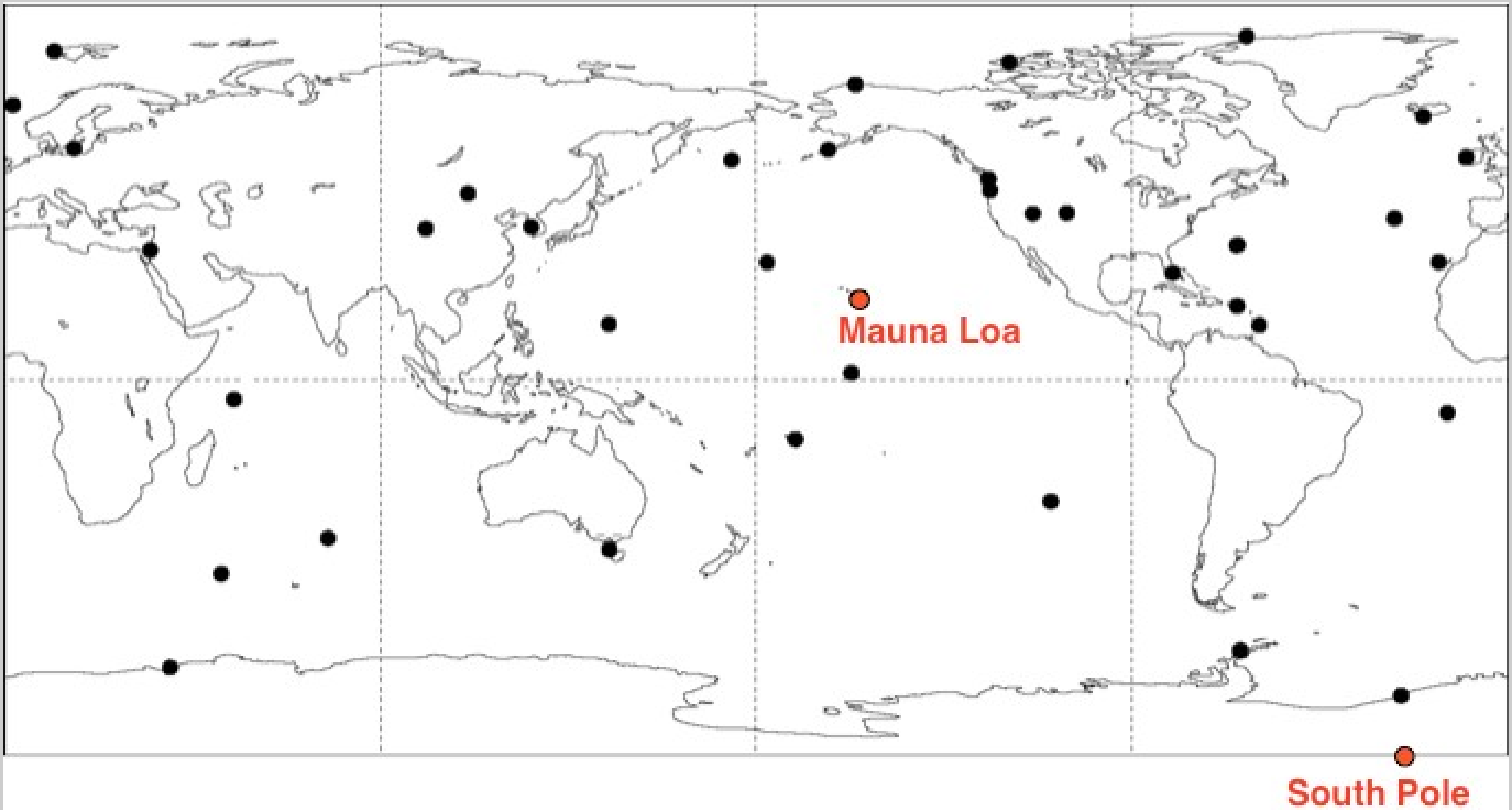
- Motivation
- Transport Inversion
- Underdetermined Inverse Problem, Pitfalls
- Role of Remote Sensing
- Carbon Cycle Data Assimilation System (CCDAS)

Land Uptake of CO2



C4MIP results (Friedlingstein et al. 2006)

The atmospheric CO2 flask sampling network



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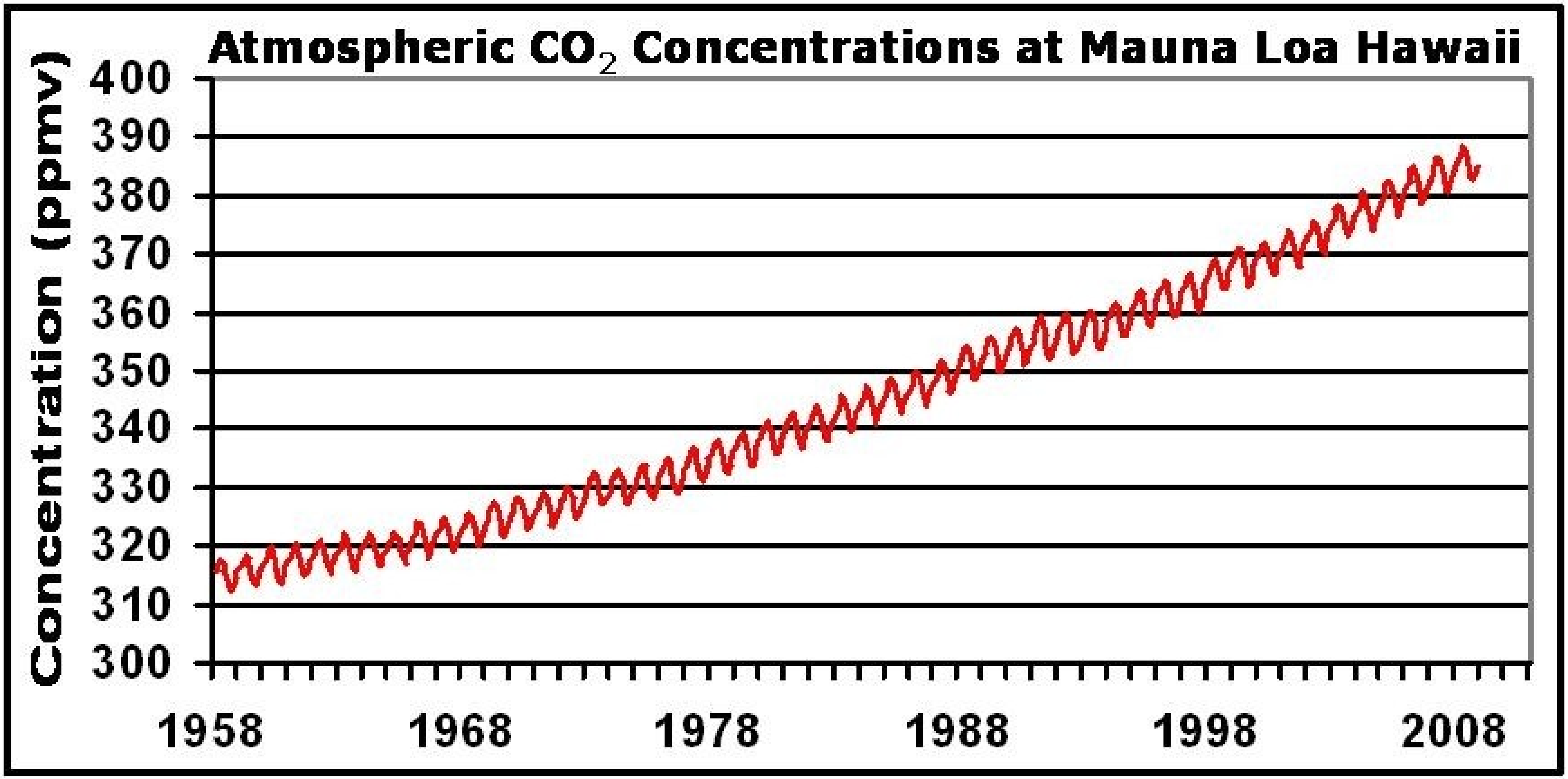
MLO


Atmospheric CO_2 Concentrations at Mauna Loa Hawaii
Concentration (ppmv)
400
390
380
370
360
350
340
330
320
310
300
1958
1968
1978
1988
1998
2008

Pioneers of Transport Inversion

- Ian Enting's group in Melbourne -> Peter Rayner
- Transport Inversion with formal computation of posterior uncertainty
- History and flavours of transport inversion:

Posterior Uncertainty, linear case

If the model is linear:

$$J(\tilde{\mathbf{x}}) = \frac{1}{2}\left[(\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d})^T \mathbf{C}(d)^{-1}(\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d}) + (\tilde{\mathbf{x}} - \mathbf{x_0})^T \mathbf{C}(x_0)^{-1}(\tilde{\mathbf{x}} - \mathbf{x_0})\right]$$

and data + priors have Gaussian PDF, then the posterior PDF is also Gaussian:

$$\rho(x) \sim e^{-J(x)}$$

with mean value:

$$\mathbf{x} = \mathbf{x_0} + [\mathbf{M}^T \mathbf{C}(d)^{-1}\mathbf{M} + \mathbf{C}(x_0)^{-1}]^{-1}\mathbf{M}^T \mathbf{C}(d)^{-1}(\mathbf{d} - \mathbf{M}\mathbf{x_0})$$

and uncertainty:

$$\mathbf{C}(x)^{-1} = \mathbf{M}^T \mathbf{C}(d)^{-1}\mathbf{M} + \mathbf{C}(x_0)^{-1}$$

which are related to the Hessian of the cost function:

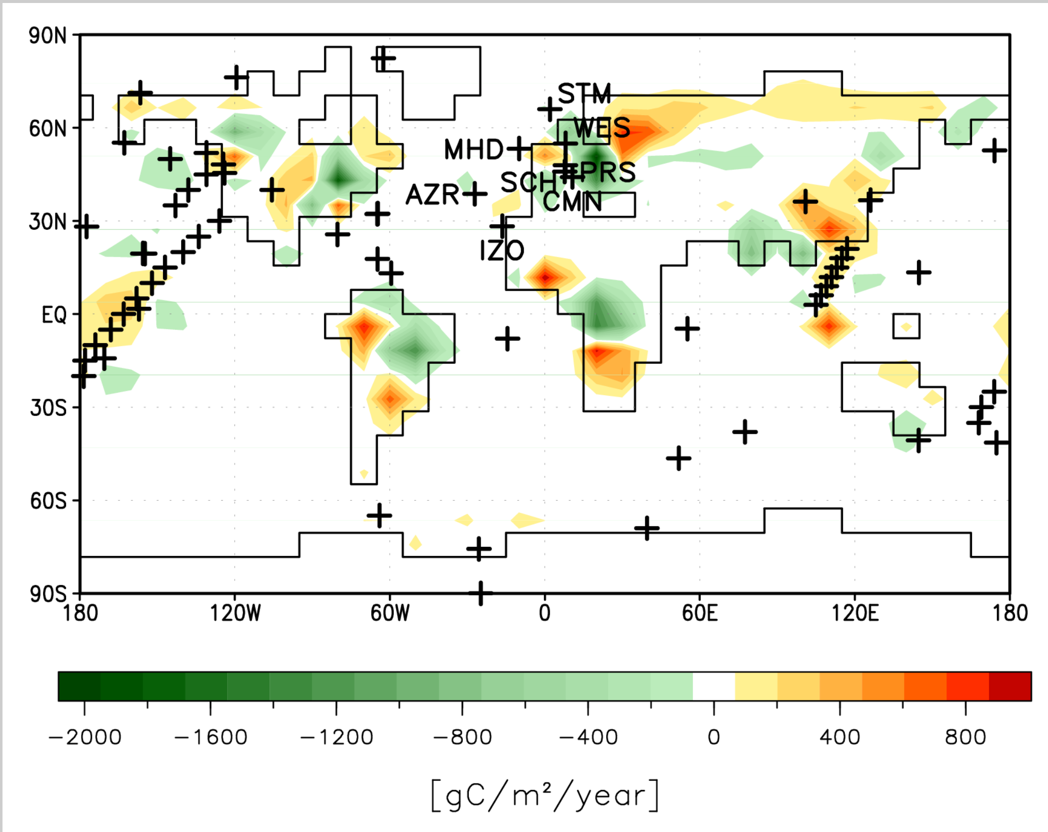
$$\mathbf{C}(x)^{-1} = \mathbf{H}$$

$$\frac{\partial^2 J}{\partial x_i \partial x_j}$$

For a non-linear model, this is an approximation

Transport Inversion

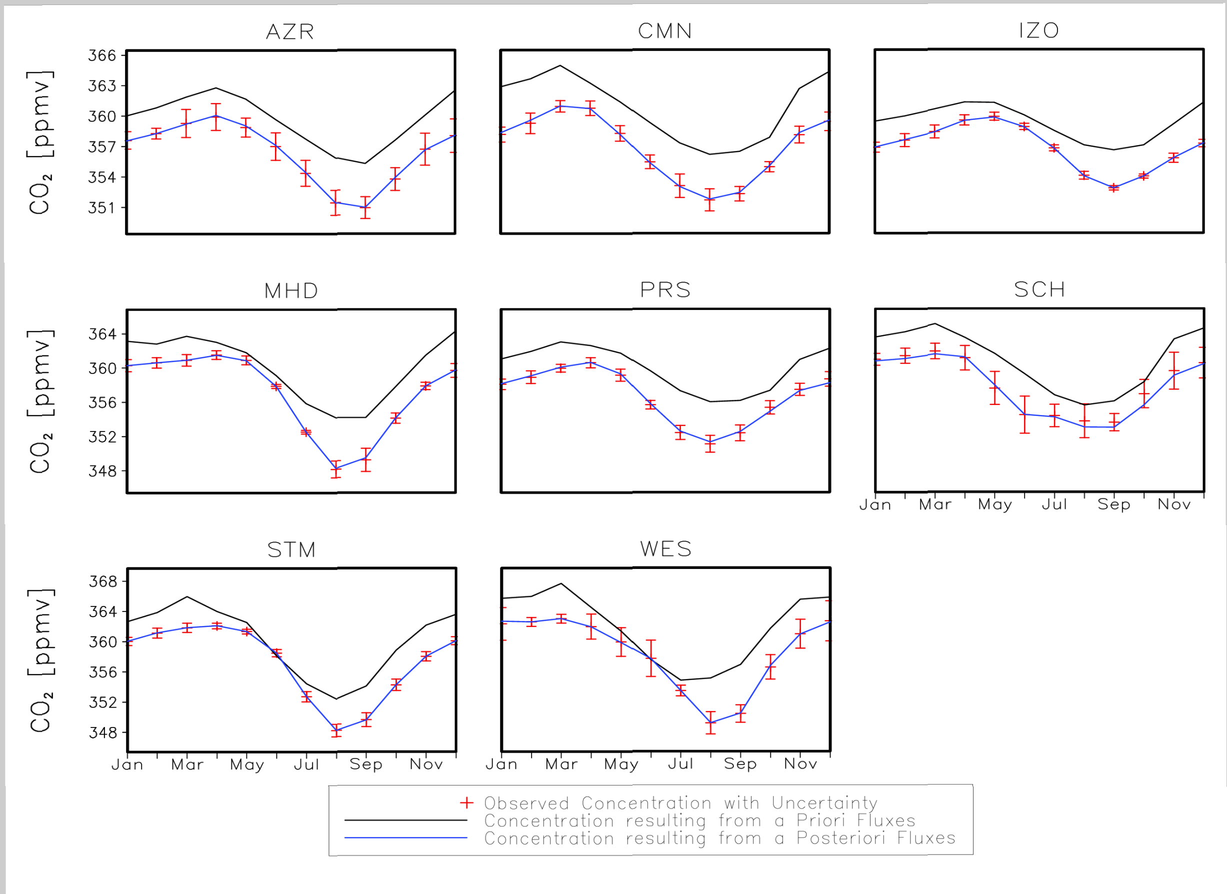
Surface flux field with a 2PgC/year sink over Europe/Hamburg ...



Kaminski and Heimann (2001)

Transport Inversion

... fully consistent with atmospheric flask sampling network

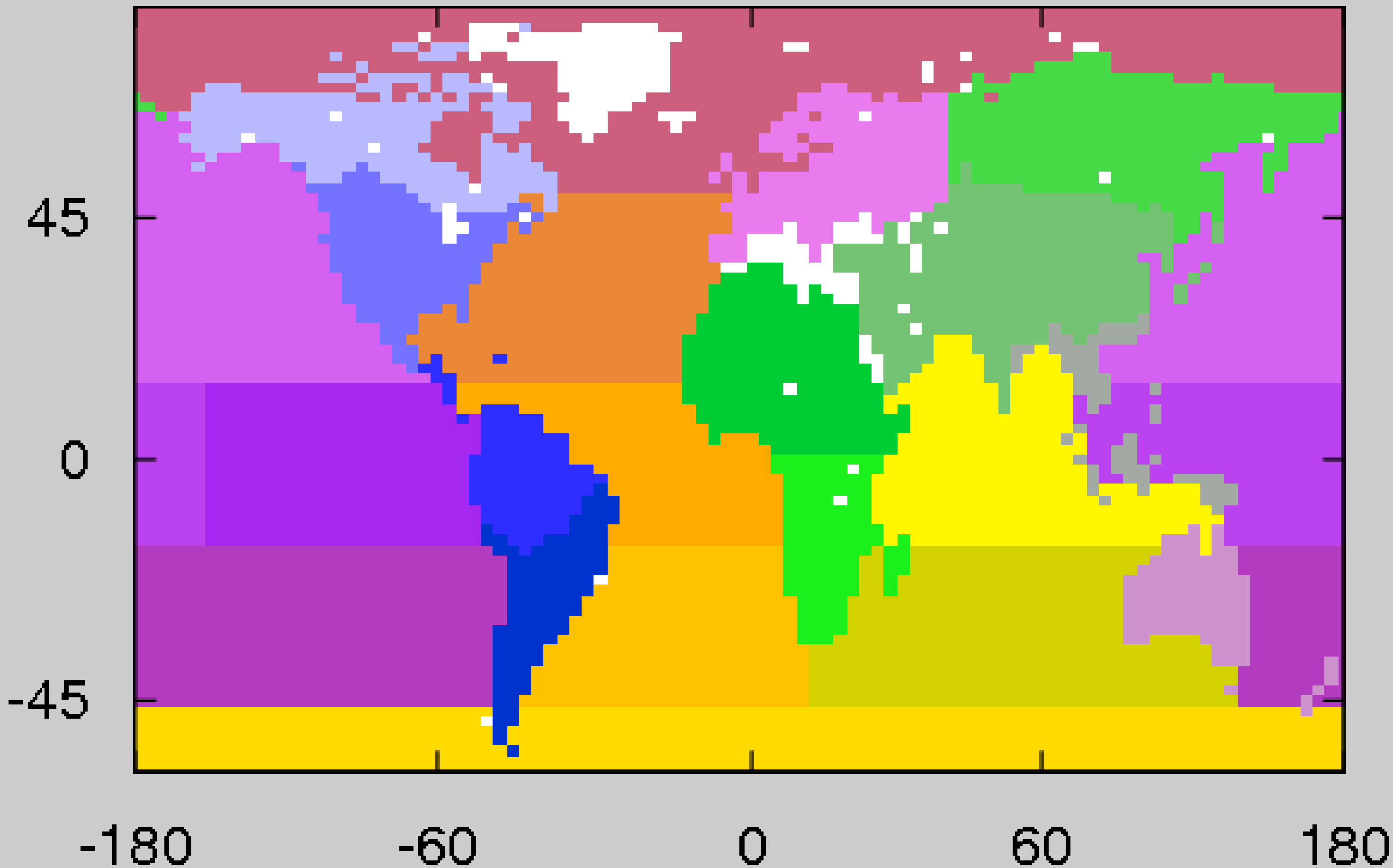


Kaminski and Heimann (2001)

Transport Inversion

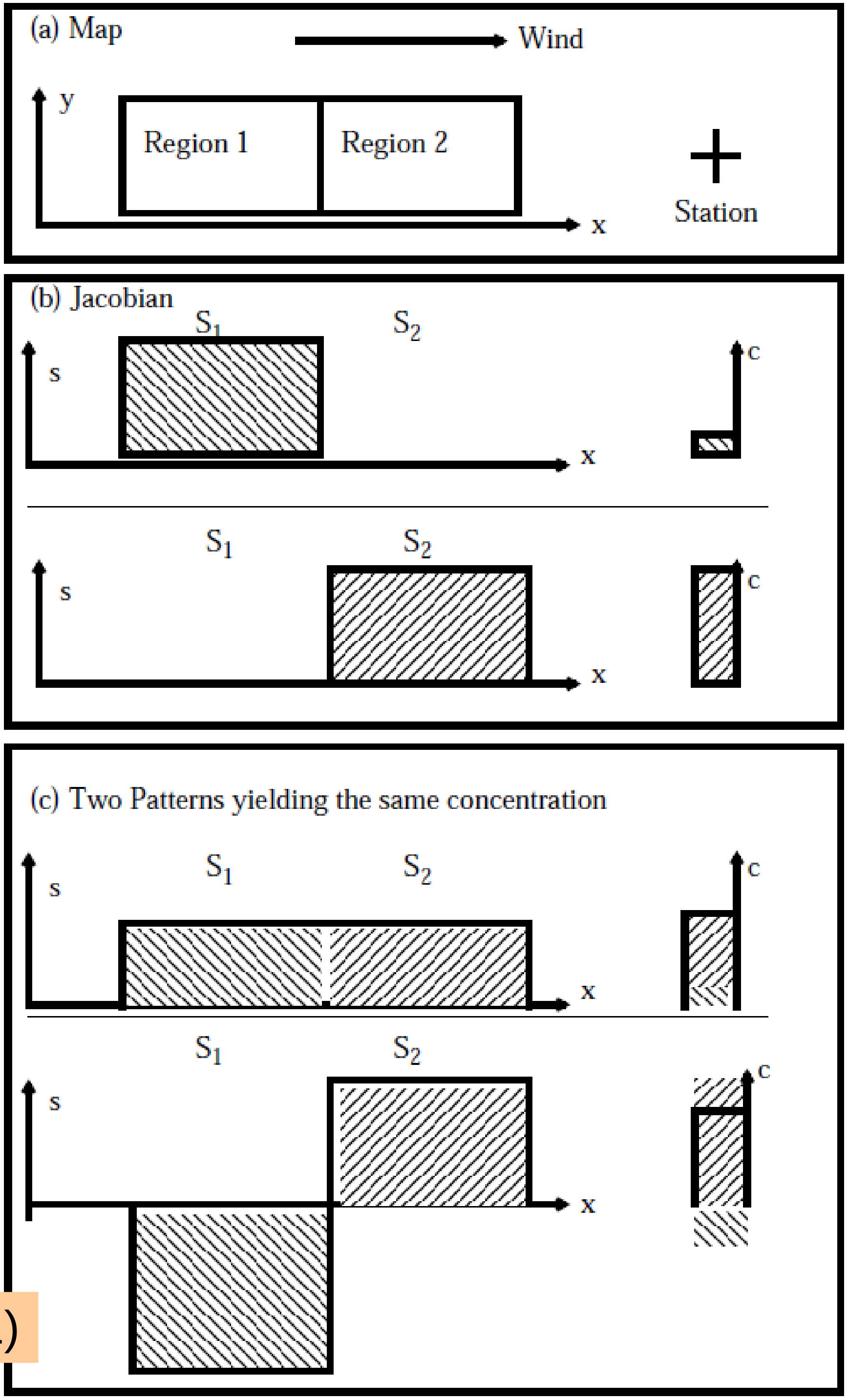
- Cannot infer 2 d flux field from point observations
- Can “hide” arbitrary sink in null space of transport inversion
- Underdetermined inverse problem needs to be stabilised by extra information, which determines solution to large extent

Extra Information through prescibed flux patterns, or spatial correlations



Aggregation Error

- Flux error can have same magnitude as flux
- Should solve for flux on full model grid
- Computationally challenging



Kaminski et al. (2001)

More Data

GEOPHYSICAL RESEARCH LETTERS, VOL. 28, NO. 1, PAGES 175-178, JANUARY 1, 2001

The utility of remotely sensed CO_2 concentration data in surface source inversions

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Abstract. This paper aims to establish the required precision for column-integrated CO_2 concentration data to be useful in constraining surface sources. We use the method of synthesis inversion and compare the uncertainties in regional sources calculated from a moderate-sized surface network and either global or oceanic coverage of column-integrated pseudodata. With a simple measure of total uncertainty, we require precision of monthly averaged column data better than 2.5 ppmv on a 8° × 10° footprint for comparable performance with the existing surface network. If coverage is only oceanic we require 1.5 ppmv precision. We recommend more detailed studies on the feasibility of obtaining such observations from current and future satellite instruments.

In this paper we aim t
integrated data may be u
tivation is the suggestion
although imprecisely, at l
ing satellite missions [e.g.

There are several pote
formation on atmospheric
For example we can mea
bands of CO_2. Similar tec
CH_4 and O_3 [see for exam
al. 1995]. The technique r
temperature profile and
H_2O) are known. Covera

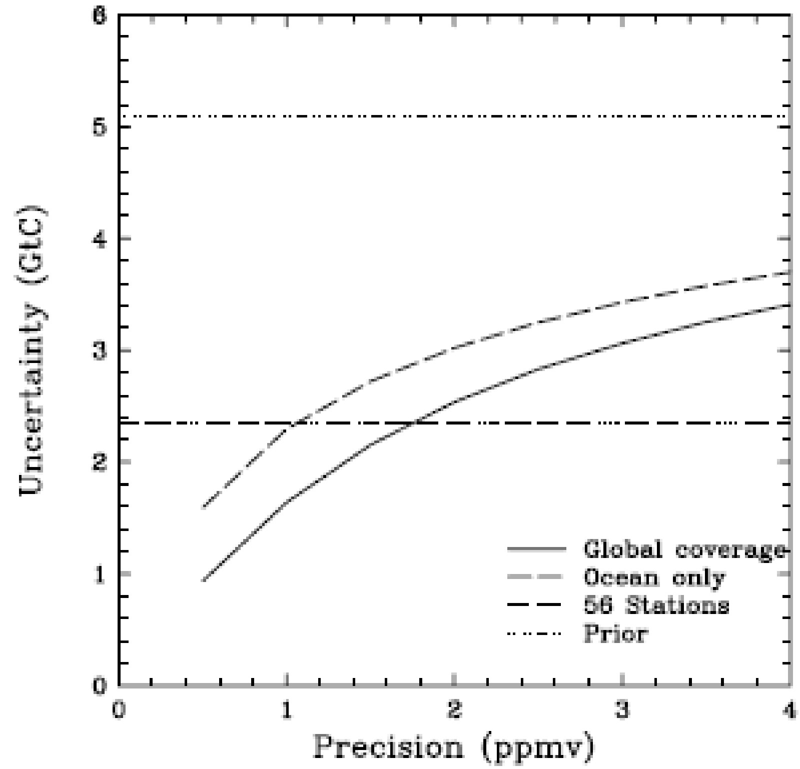

Figure 3. Plot of "total uncertainty" (see Eq. 1) in GtC yr^{-1} against the precision (ppmv) of column-integrated data. The dotted horizontal line shows the prior uncertainty while the dash-dot horizontal line shows the case for the surface network of Fig. 1.

-> more in MB's lecture

Posterior Uncertainty, linear case

$$J(\tilde{\mathbf{x}}) = \frac{1}{2}\left[(\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d})^T \mathbf{C}(d)^{-1}(\mathbf{M}\tilde{\mathbf{x}} - \mathbf{d}) + (\tilde{\mathbf{x}} - \mathbf{x_0})^T \mathbf{C}(x_0)^{-1}(\tilde{\mathbf{x}} - \mathbf{x_0})\right]$$

If the model is linear:

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with mean value:

$$\mathbf{x} = \mathbf{x_0} + [\mathbf{M}^T \mathbf{C}(d)^{-1}\mathbf{M} + \mathbf{C}(x_0)^{-1}]^{-1}\mathbf{M}^T \mathbf{C}(d)^{-1}(\mathbf{d} - \mathbf{M}\mathbf{x_0})$$

and uncertainty:

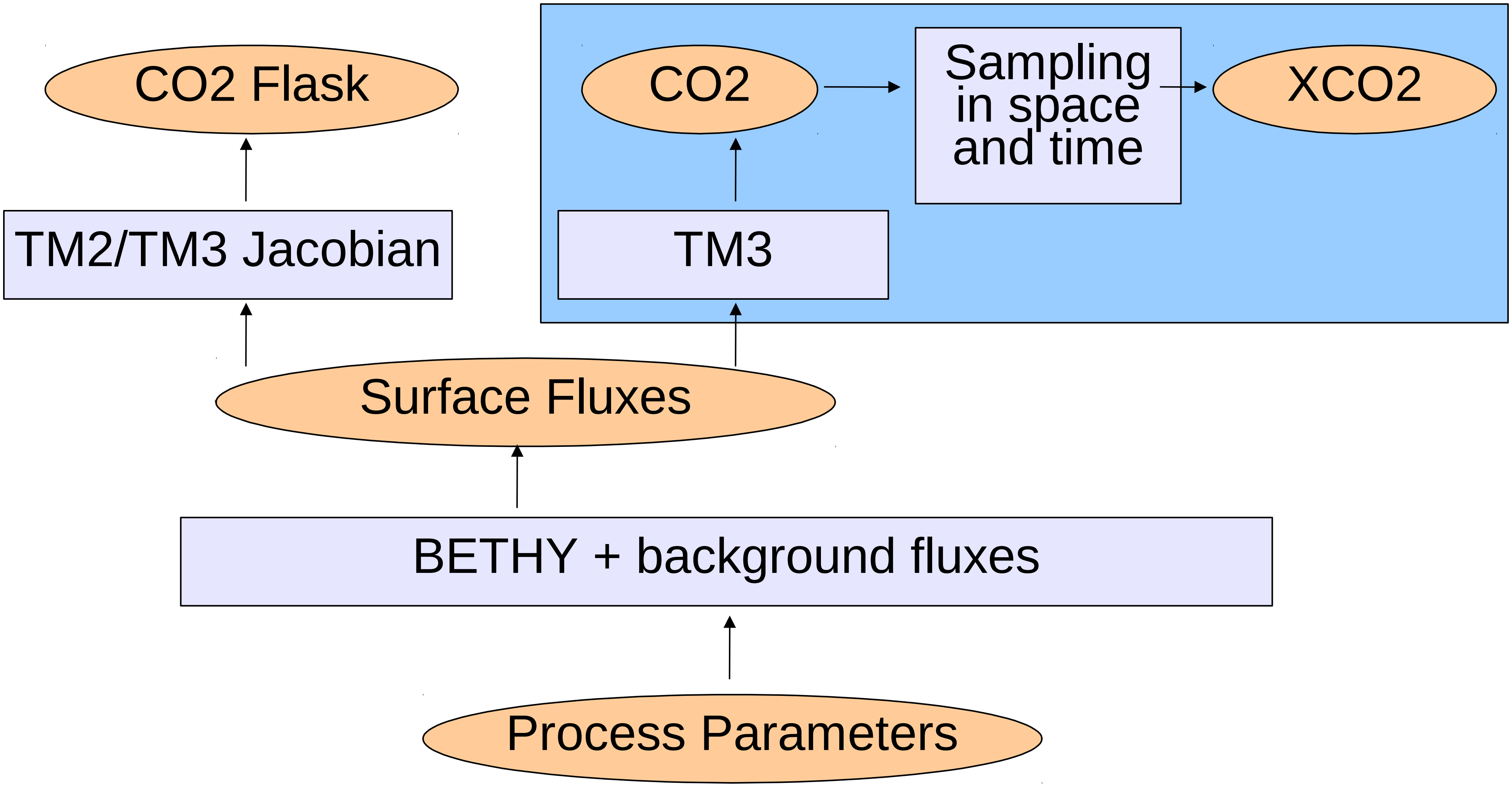
$$\mathbf{C}(x)^{-1} = \mathbf{M}^T \mathbf{C}(d)^{-1}\mathbf{M} + \mathbf{C}(x_0)^{-1}$$

which are related to the Hessian of the cost function:

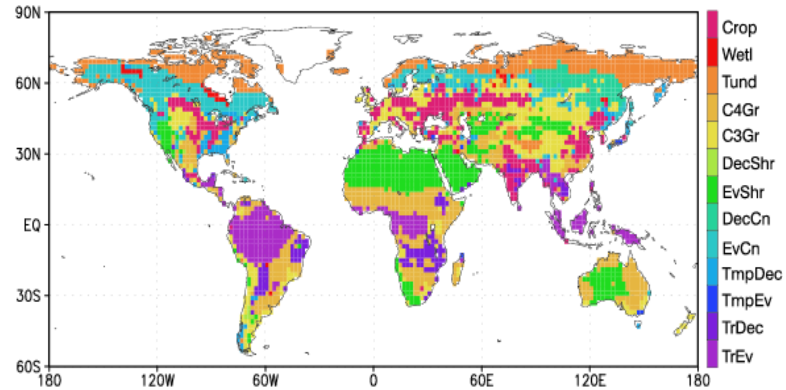
$$\mathbf{C}(x)^{-1} = \mathbf{H} \qquad \frac{\partial^2 J}{\partial x_i \partial x_j}$$

For a non-linear model, this is an approximation

CCDAS-Processing Chain

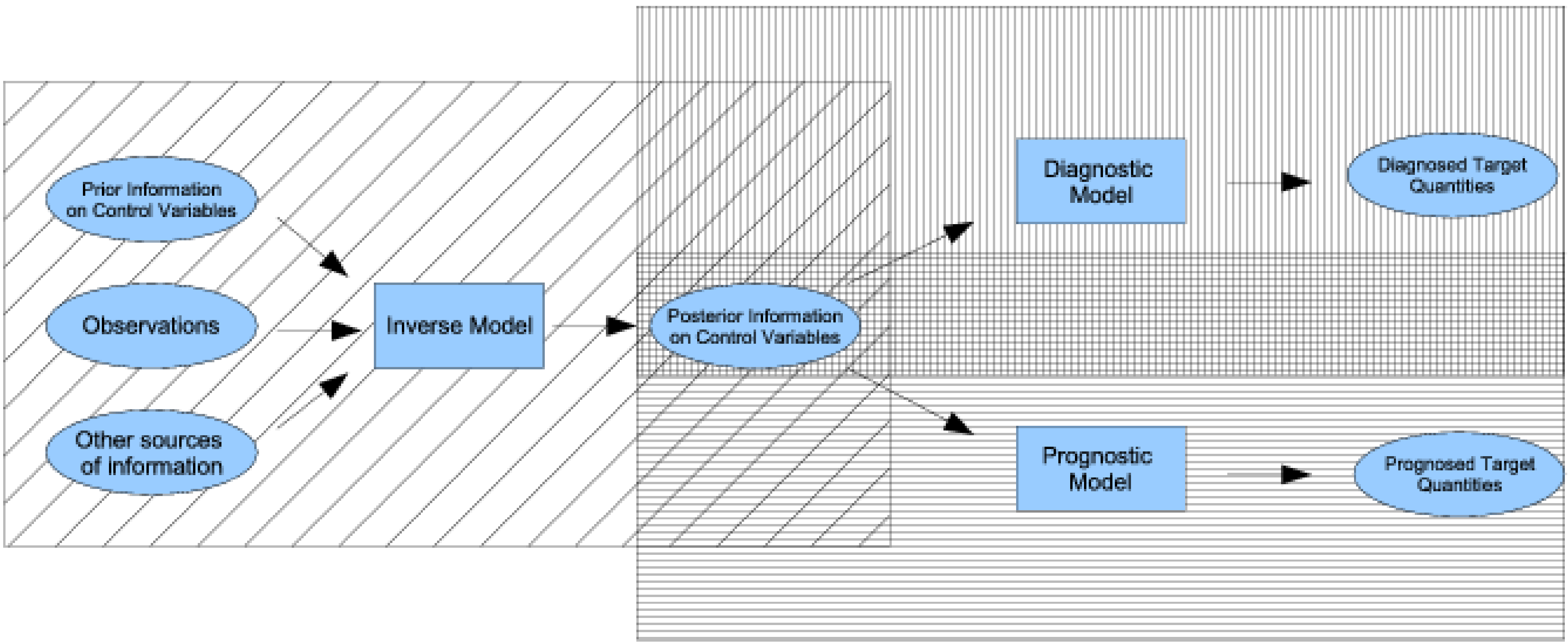


BETHY setup



- 2 by 2 degrees, global
- 13 Plant Functional Types
- 57 Process Parameters (Initial Conditions)
  - 3 PFT specific
  - 17 not PFT specific
  - 1 atmospheric
- Parameter values taken from Scholze et al. (2007)
- Prior parameter uncertainties taken from Scholze et al. (2007), in the order of 25%

CCDAS scheme



Scholze et al. (2007)

Uncertainty calculation in 2 steps

x: Parameters

x_{pr}: Priors

C_{pr}: Uncertainties

$M(x)$: Model

d: Observations

C_d: Their uncertainties

σ_{d_i}: Uncorrelated!

$J(x)$: Cost function

$\frac{d^2 J(x)}{dx^2}$: Hessian

x_{po}: Posterior parameters

C_{po}: Posterior uncertainties

$y(x)$: Target quantity

σ_y: Its uncertainty

Inverse step:

$$J(x) = \frac{1}{2}(x - x_{pr})^T C_{pr}{}^{-1}(x - x_{pr}) + \frac{1}{2}\sum_{i=1,nd}\left(\frac{M_i(x) - d_i}{\sigma_{d_i}}\right)^2$$

$$\frac{d^2 J(x)}{dx^2} = C_{pr}{}^{-1} + \sum_{i=1,nd}\frac{1}{\sigma_{d_i}^2}\frac{d^2}{dx^2}(M_i(x) - d_i)^2$$

uncertainty in observations AND model

- Hessian independent of x for linear model
- For synthetic data use $d = M(x)$.
- Decomposes nicely, can precompute model contribution

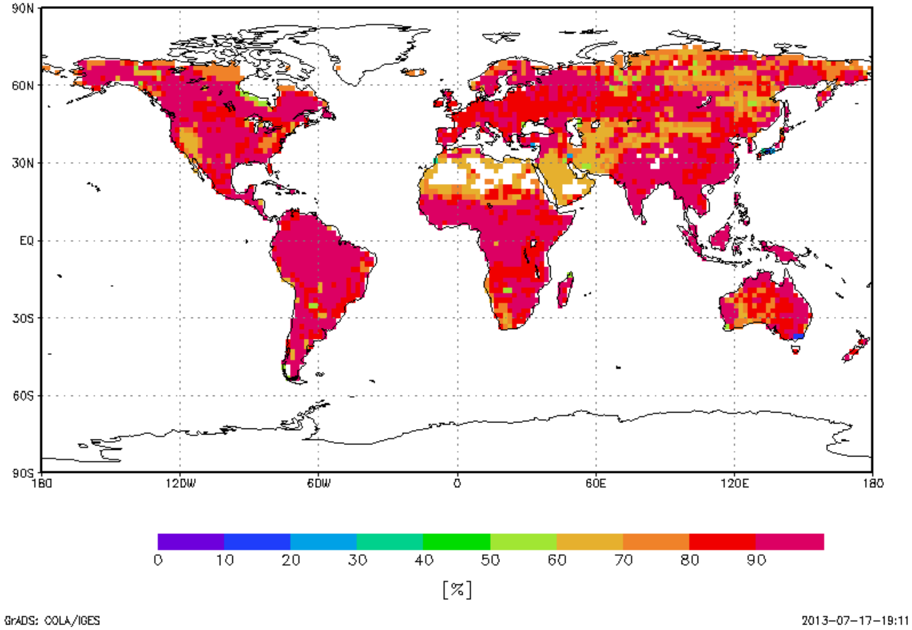
$$C_{po} \approx \frac{d^2 J(x_{po})}{dx^2}^{-1}$$

Propagation step:

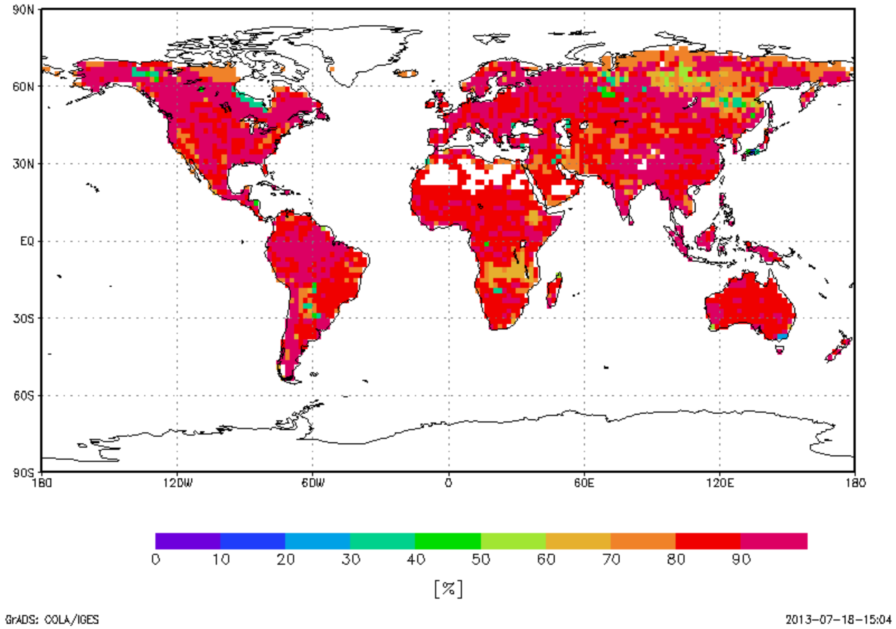
$$\sigma_y^2 \approx \frac{dy(x_{po})}{dx} C_{po} \frac{dy(x_{po})}{dx}^T \approx \frac{dy(x_{po})}{dx}\frac{d^2 J(x_{po})}{dx^2}^{-1}\frac{dy(x_{po})}{dx}^T$$

All derivative code generated from model code by automatic differentiation tool TAF

BESD biased vs flask samples

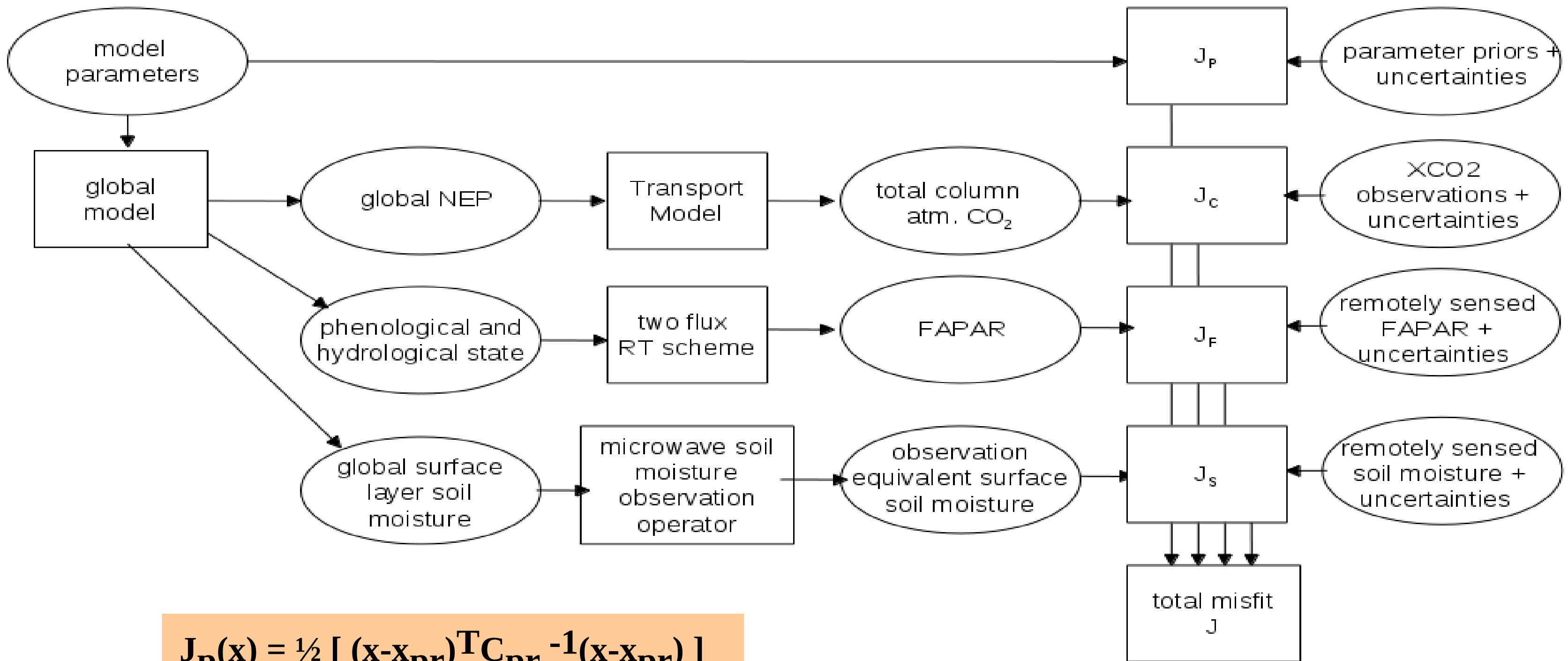


BESD (Reuter et al., 2001)
one year



flask samples
20 years

CCDAS can integrate multiple data streams



$J_p(x) = ½ [(x-x_{pr})^T C_{pr}^{-1} (x-x_{pr})]$

$J_d(x) = ½ [(M(x)-d)^T C_d^{-1} (M(x)-d)]$

Consistent Assimilation :
- **all data streams jointly**
- **in a single long assimilation window**

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