Validation of FVM Solver for Atmospherics Flows: Lee Waves

Viktor Šíp

Department of Technical Mathematics, Faculty of Mechanical Engineering, Czech Technical University

INTRODUCTION

Micro- to mesoscale CFD simulations of planetary boundary layer flows are indespensable complement to macroscale models. They are applicable in a variety of problems, including urban flow and polution dispersion. Here we present the description of currently developed finite volume solver for such applications and its validation on a problem of moutain lee waves in stratified atmosphere.

NUMERICAL MODEL DESCRIPTION

Simplifications

Several physical simplifications were used:

1. We assume that the flow is incompressible. Such assumption is usually reasonable for domains with height $H < 10$ km.
2. We are interested only in steady state solutions, not transient behaviour.
3. Coriolis force is ommited.
4. Moisture effects are neglected.

Equations

We solve continuity equation, equations for velocity components and for potential temperature. Artificial compressibility with parameter β is utilized. Pressure, density and potential temperature are splitted into background component in hydrostatic balance and fluctuations:

$$p = p_0 + p', \rho = \rho_0 + \rho', \theta = \theta_0 + \theta',$$

and the set of RANS equations is as follows:

$$p'_t/\beta + (u_1)_x + (u_2)_y + (u_3)_z = 0$$

$$(u_1)_t + (u_1^2 + p'/\rho_0)_x + (u_2 u_1)_y + (u_3 u_1)_z = \nu(u_{1xx} + u_{1yy} + u_{1zz})$$

$$(u_2)_t + (u_1 u_2)_x + (u_2^2 + p'/\rho_0)_y + (u_3 u_2)_z = \nu(u_{2xx} + u_{2yy} + u_{2zz}) + g\frac{\theta'}{\theta_0}$$

$$(u_3)_t + (u_1 u_3)_x + (u_2 u_3)_y + (u_3^2 + p'/\rho_0)_z = \nu(u_{3xx} + u_{3yy} + u_{3zz})$$

$$\theta_t + (u_1\theta)_x + (u_2\theta)_y + (u_3\theta)_z = k/c_v\left((\theta_x/\rho)_x + (\theta_y/\rho)_y + (\theta_z/\rho)_z\right)$$

Numerical method

Finite volume method on unstructured grid is used. Numerical flux AUSM$^+$-up [3], designed for flows at all speed regimes, is employed for convective fluxes evaluation.

Second order accuracy in space is achieved via linear reconstruction, where gradients are evaluated by means of least squares approach. Venkatakrishnan limiter [4] is used to prevent unphysical oscillations.

We use implicit second order BDF method with variable timestep for temporal discretization:

$$\frac{1 + 2\rho_n}{1 + \rho_n}\boldsymbol{W}^{n+1} - (1 + \rho_n)\boldsymbol{W}^n + \frac{\rho_n^2}{1 + \rho_n}\boldsymbol{W}^{n-1} = \Delta t_n f(\boldsymbol{W}^{n+1}),$$

Here $\boldsymbol{W}^n$ is a state vector at time step n and $\rho_n = \Delta t_n / \Delta t_{n-1}$ is a ratio of consequent time step lengths.

Resulting system of nonlinear equations is solved by Newton's method. Condition number of nested linear systems is reduced by the usage of ILU preconditioner. The precondition matrix is recalculated every 20th time step (or after linear system solver failure) to lower the computational costs.

Turbulence modelling

Simple algebraic mixing-length model is used. Following the Boussinesq assumption we consider the viscosity to be a sum of laminar and turbulent viscosity, $\nu = \nu_{lam} + \nu_{turb}$, where

$$\nu_{turb} = l^2 \sqrt{\left(\frac{\partial u_1}{\partial x_2}\right)^2 + \left(\frac{\partial u_3}{\partial x_2}\right)^2}$$

and l is a mixing length according to [2].

NUMERICAL EXPERIMENTS

Lee waves

Problem of lee waves (ie. internal gravity induced waves downstream of an obstacle) in stratified flow is a problem well studied from all viewpoints: observational, experimental, theoretical as well as numerical. (For comprehensive study, see eg. [1].) As such, it can serve as a good validation case for micro- to mesoscale models.

Case setup

Here we study stratified flow over 2D hill (infinite moutain ridge) of 'Witch of Agnesi' shape, $y(x) = h/((1 + x/a)^2)$, with height $h = 600$ m and half-width $a = 400$ m.

Domain itself has size 20 km x 10 km.

Boundary conditions:

- Inlet velocity: log wind profile,
- Inlet temperature: linear profile with $T_0 = 293.15$ K and $\frac{\partial T}{\partial y} = 0.01$ K/m,
- Inlet and outlet pressure: barometric formula,
- No-slip condition at the bottom,
- Prescribed pressure, temperature and velocity at the top, matching the inlet BC.

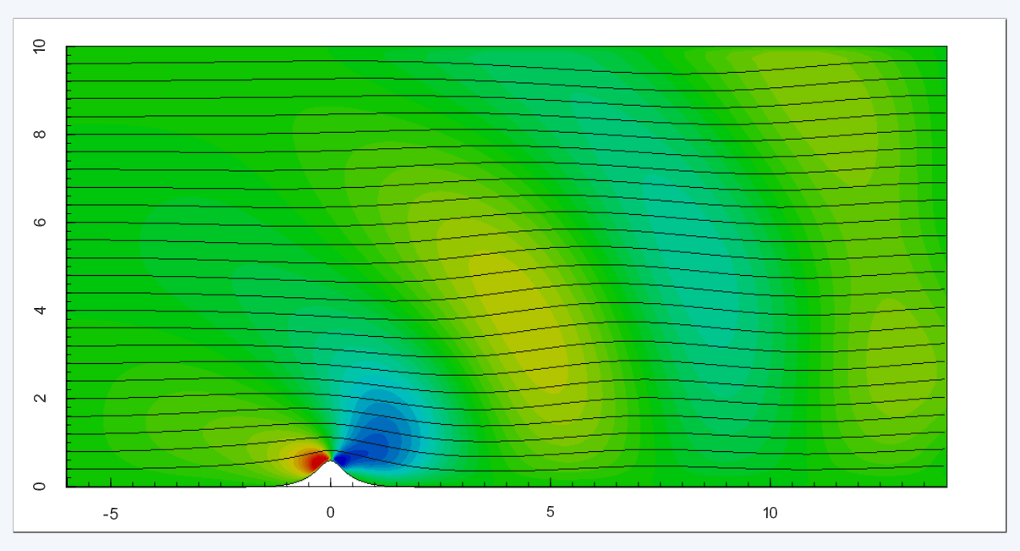
Given temperature profile results in a strong stable stratification with Brunt-Väisälä frequency

$$N = \sqrt{\frac{g}{\theta}\frac{\partial\theta}{\partial y}} = 0.0256\text{s}^{-1}$$

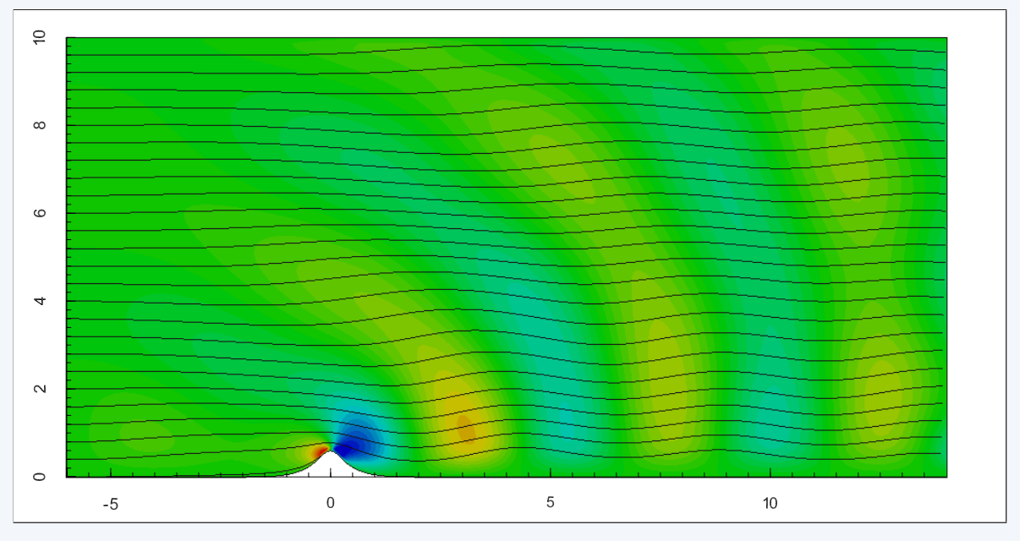
at the ground level.

We simulated five cases with velocity U ranging from 31 m/s to 10 m/s (Nh/U from 0.5 to 1.5) and three cases with velocity from 7.7 m/s to 1.9 m/s (Nh/U from 2.0 to 8.0), which are not presented here (see Discussion for reason why).

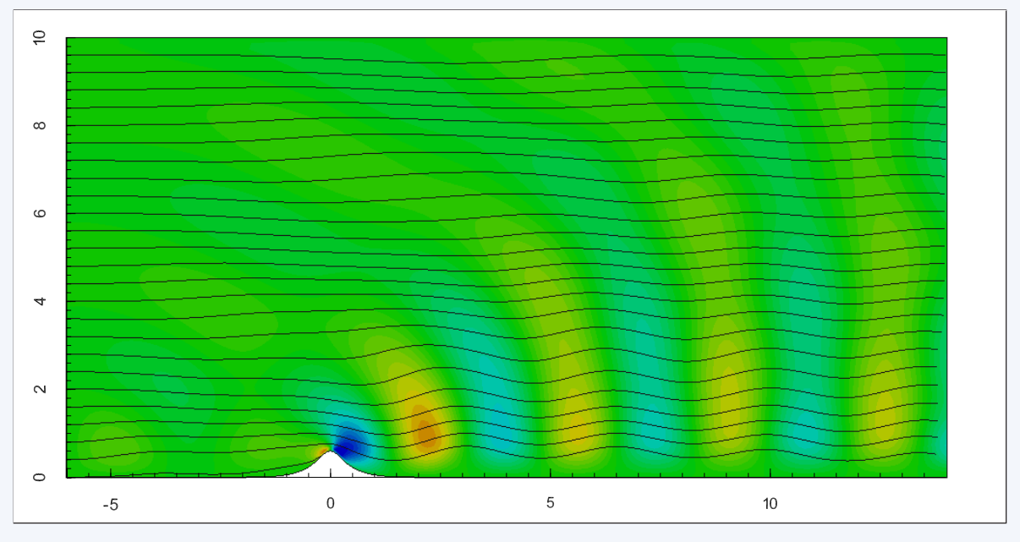
All simulations were performed on a computational grid having 240x100 cells.



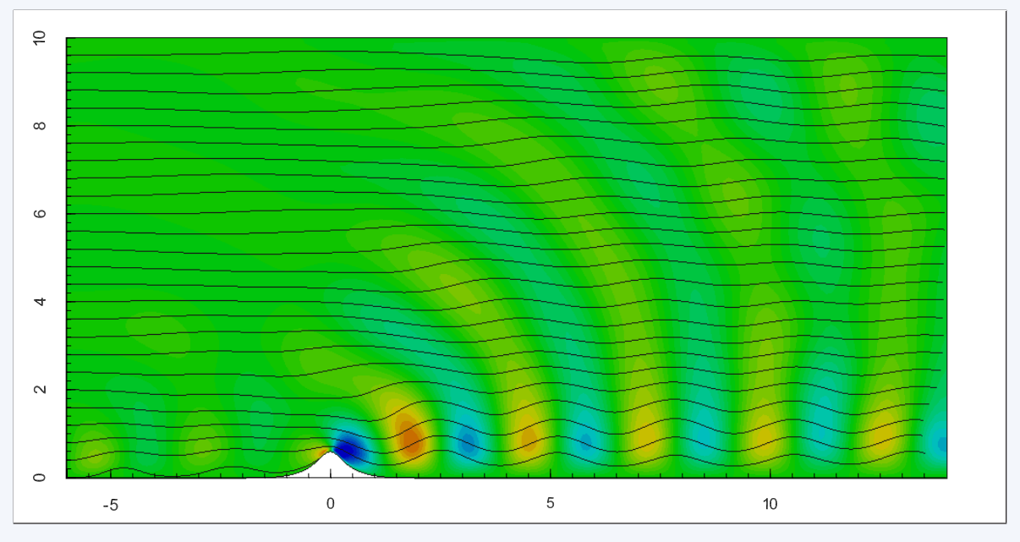
(a) $Nh/U = 0.5$



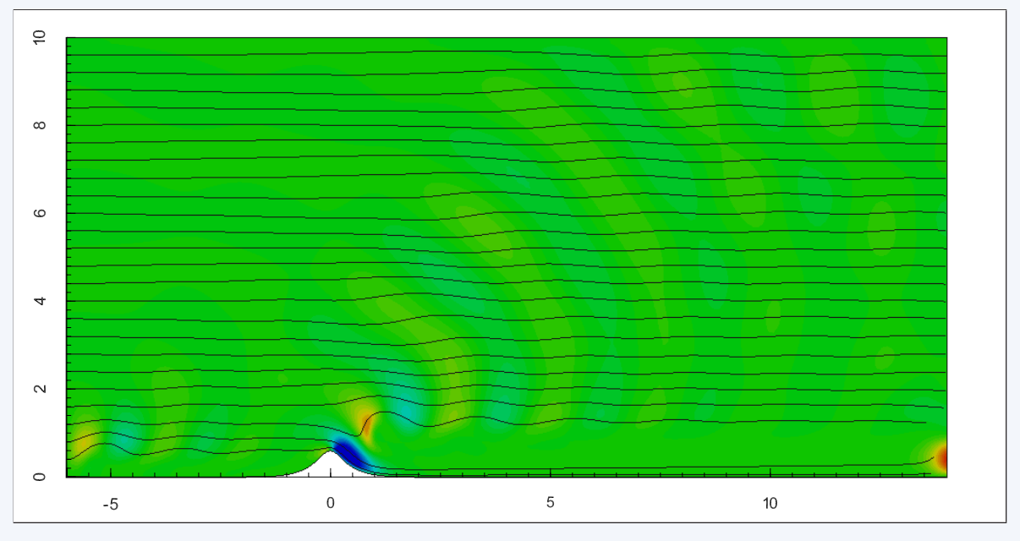
(b) $Nh/U = 0.75$



(c) $Nh/U = 1.0$



(d) $Nh/U = 1.25$



(e) $Nh/U = 1.5$

Figure 1: Velocity streamlines for varying Nh/U. Background color by y-component of velocity.

Results

We compared the predicted length of lee waves $\lambda_t = \frac{2\pi U}{N}$ with the length of the waves from the simulation λ_{calc}.

Nh/U	U	Na/U	λ_t	λ_{calc}
0.5	30.72 m/s	0.33	7540 m	7500 m
0.75	20.48 m/s	0.5	5027 m	4500 m
1.0	15.36 m/s	0.67	3770 m	3300 m
1.25	12.29 m/s	0.84	3016 m	2650 m
1.5	10.24 m/s	1.0	2513 m	2100 m

Good agreement can be seen for case with $Nh/U = 0.5$, in all other cases the difference of 400 - 500 m is present. The reason of this difference is yet unclear.

Case $Nh/U = 1.5$ deserves special attention, as we can see a change of the flow character. Upstream flow is blocked by the mountain, while strong downslope winds develop downstream. Smaller waves can be observed in the upper layer.

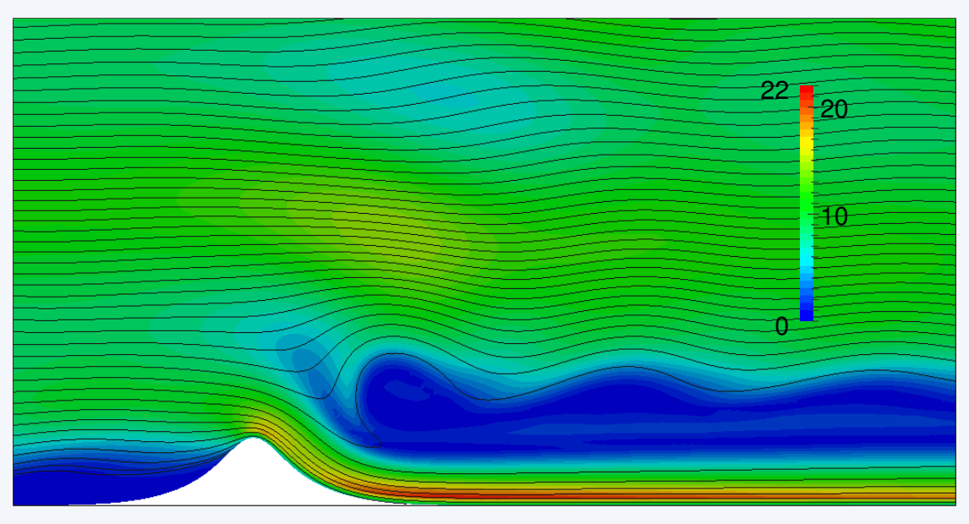


Figure 2: Hill detail for $Nh/U = 1.5$. Velocity streamlines, background color by velocity magnitude.

Discussion

Present investigation proved the ability of the method to replicate lee waves in regimes with smaller values of Nh/U. However, the computations failed in cases where $Nh/U > 2.0$. The flow dynamics in these regimes is more complex, with phenomena of hydraulic jump, wave breaking and downslope rotors appearing, as partially affirmed by the case of $Nh/U = 1.5$. Simulation of these features proved to be more challenging and will be the object of further study.

REFERENCES

[1] Peter G. Baines. *Topographic Effects in Stratified Flows*. Cambridge University Press, 1995.

[2] Alfred K. Blackadar. The vertical distribution of wind and turbulent exchange in a neutral atmosphere. *Journal of Geophysical Research*, 67(8):3095–3102, 1962.

[3] Meng-Sing Liou. A sequel to AUSM, part II: AUSM+-up for all speeds. *Journal of Computational Physics*, 214:137–170, 2006.

[4] V. Venkatakrishnan. Convergence to steady state solutions of the euler equations on unstructured grids with limiters. *Journal of Computational Physics*, 118:120–130, 1995.