A generalization of the SBAS approach to estimate the temporal evolution of Precipitable Water Vapour from time-series of InSAR interferograms

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ISTITUTO PER LE APPLICAZIONI DEL CALCOLO

FRINGE 2015 WORKSHOP
Advances in the Science and Applications of
SAR interferomtry and Sentinel-1 InSAR Workshop

Ciências ULisboa INSTITUTO DOM LUIZ (IDL) agenzia spaziale italiana

Outline

- Precipitable Water Vapor (PWV) and InSAR
- PWV temporal evolution by InSAR
- Short-temporal baseline approach for PWV
- B-splines and Continuous-time modelling of PWV
- Results

Precipitable Water Vapour and InSAR

Let us suppose to have an interferogram corrected for topography

$$\varphi(t_M, t_S) = \left[\psi^{def}(t_S) - \psi^{def}(t_M)\right] + \left[\psi^{atm}(t_S) - \psi^{atm}(t_M)\right]$$

where

$$\psi^{def}(t) = \frac{4\pi}{\lambda} d(t)$$

$$\psi^{atm}(t) = \frac{4\pi}{\lambda} \frac{1}{\Pi \cdot M(\vartheta)} PWV(t)$$

$$M(\vartheta) = \frac{1}{\cos\vartheta}$$ is the mapping function

$$\Pi = \frac{10^6}{\rho_{H_2O} \cdot R_v \cdot \left[\frac{k_3}{T_m} + k_2^{'}\right]}$$

Precipitable Water Vapour and InSAR

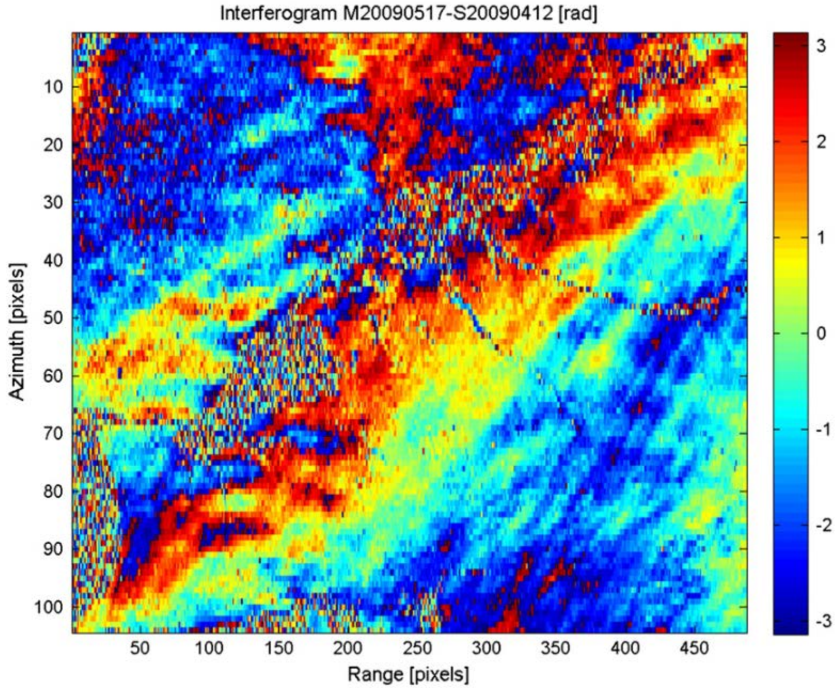
If terrain deformation can be neglected, InSAR can provide maps of PWV temporal changes

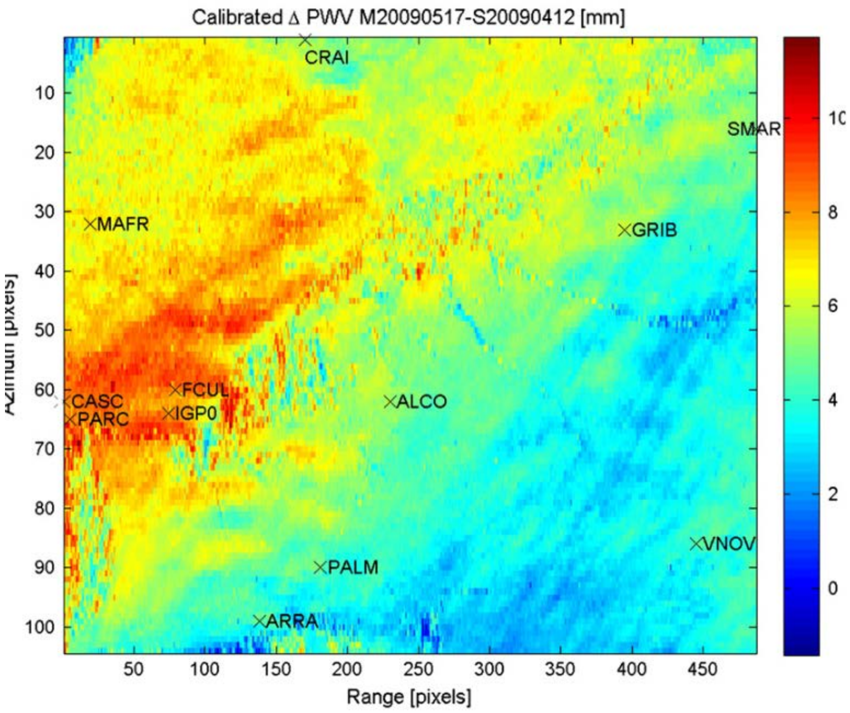
$$\Delta PWV\left(t_M, t_S\right) = \Pi \cdot \frac{\lambda}{4\pi} \cdot \Delta\varphi \cdot M_r$$

A set of independent measurements of ΔPWV made by a network of permanent GPS stations can be used to calibrate InSAR measurements. Each station measures the mean PWV in a circular area with a radius of about 3.8 km depending on the cut-off angle set in the GPS processing. The idea is to use GPS estimates of PWV at the acquisition times of master and slave SAR images to compute an independent set of ΔPWV.

Mateus et al., "Can spaceborne SAR interferometry be used to study the temporal evolution of PWV?", Atmospheric Research, 119, 70-80, 2013

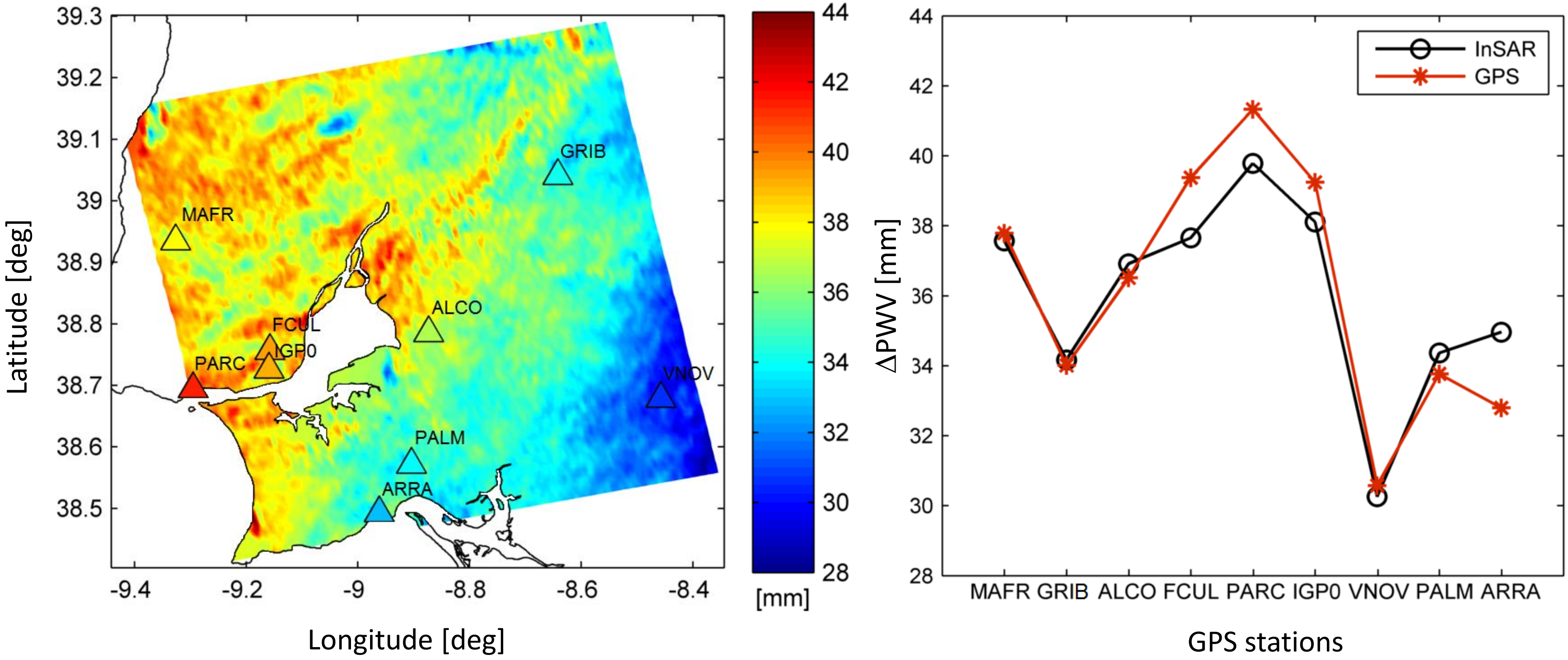
Precipitable Water Vapour and InSAR





Precipitable Water Vapour and InSAR

30/08/2009 → 04/10/2009



Precipitable Water Vapour and InSAR

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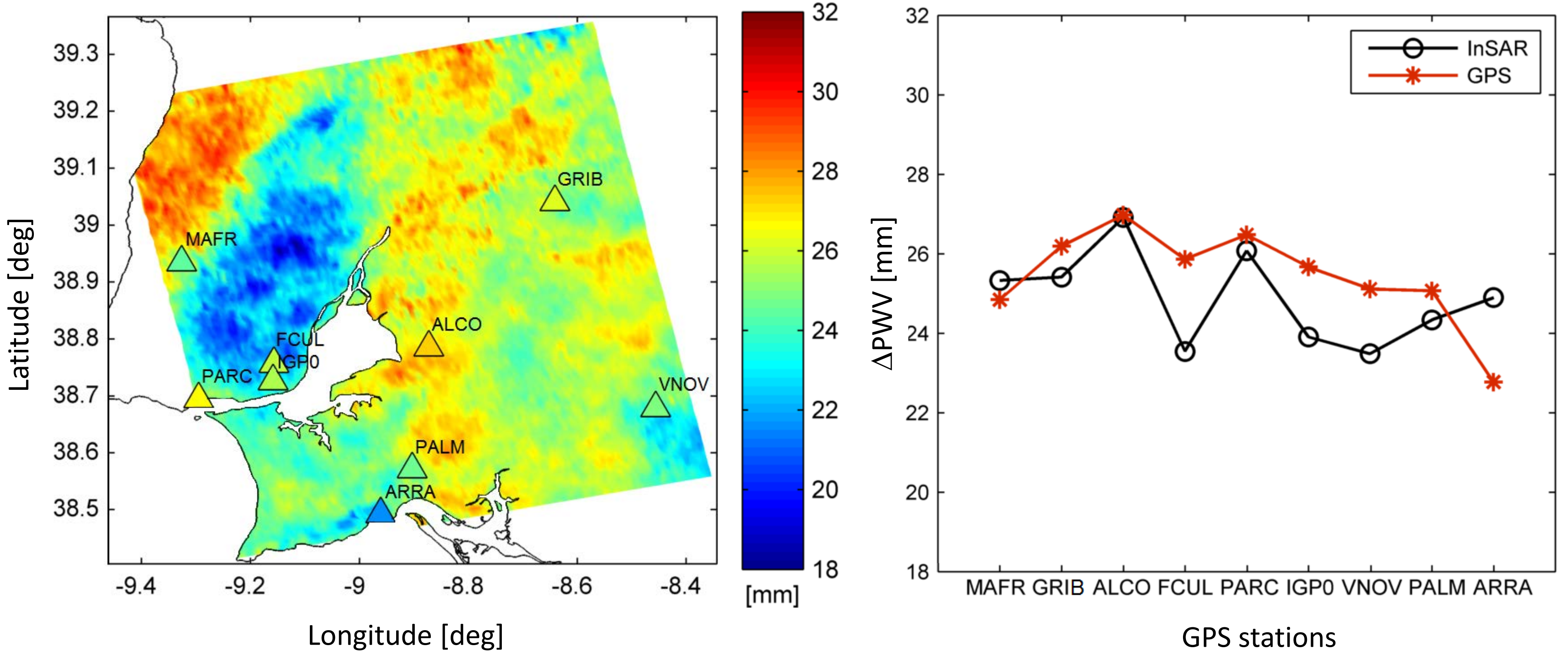
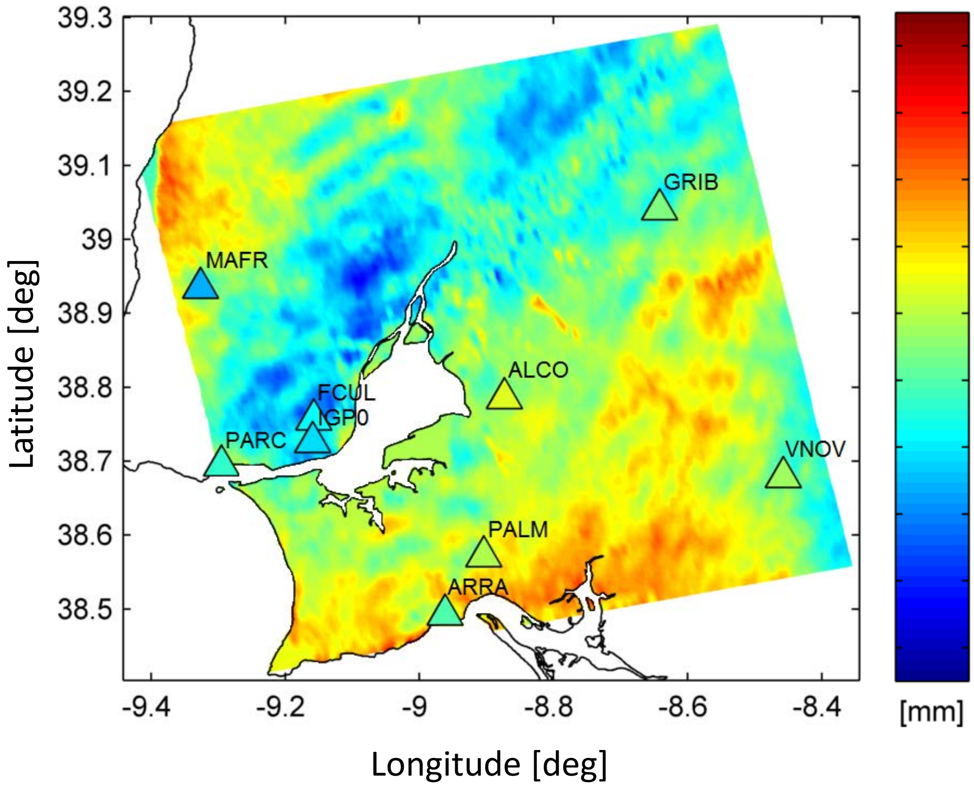
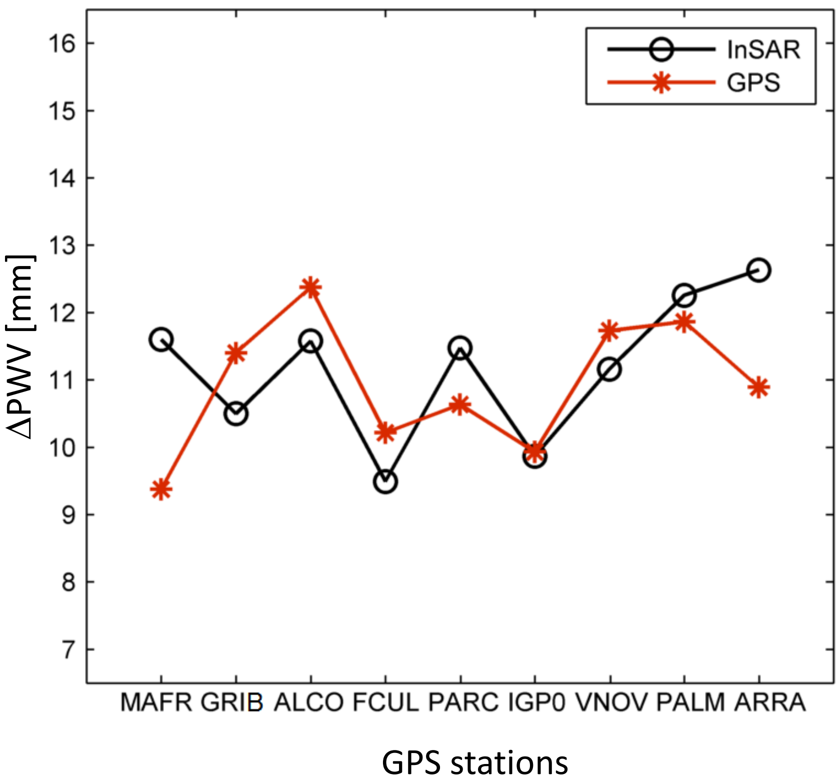


Precipitable Water Vapour and InSAR

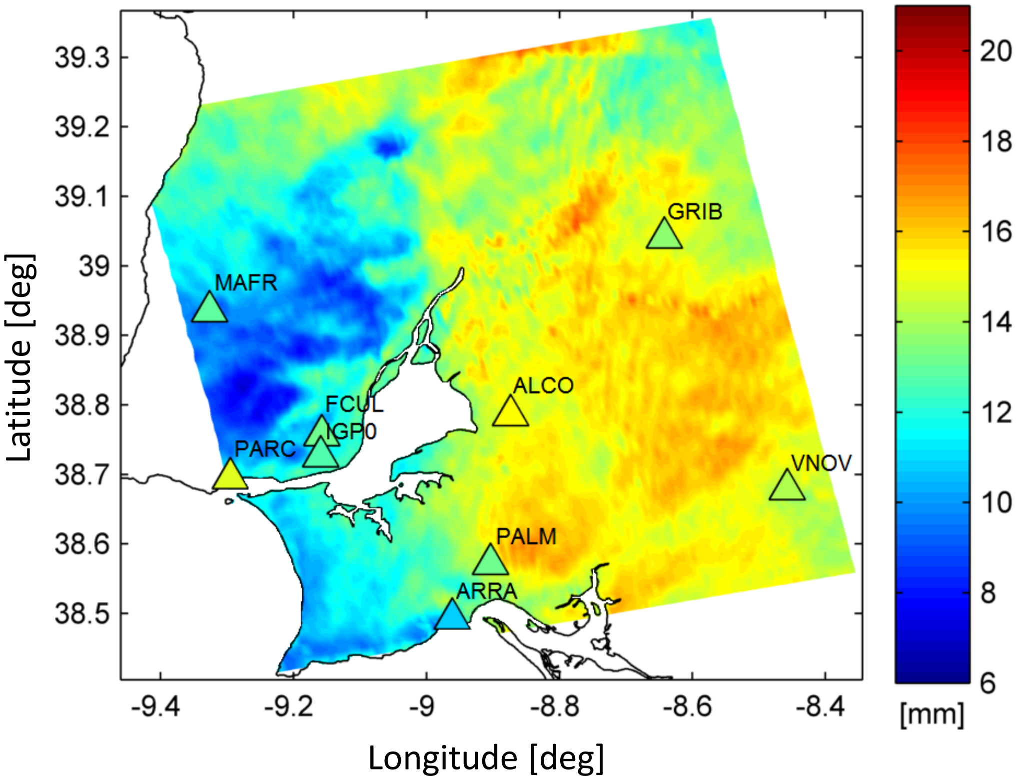
12/04/2009 → 17/05/2009

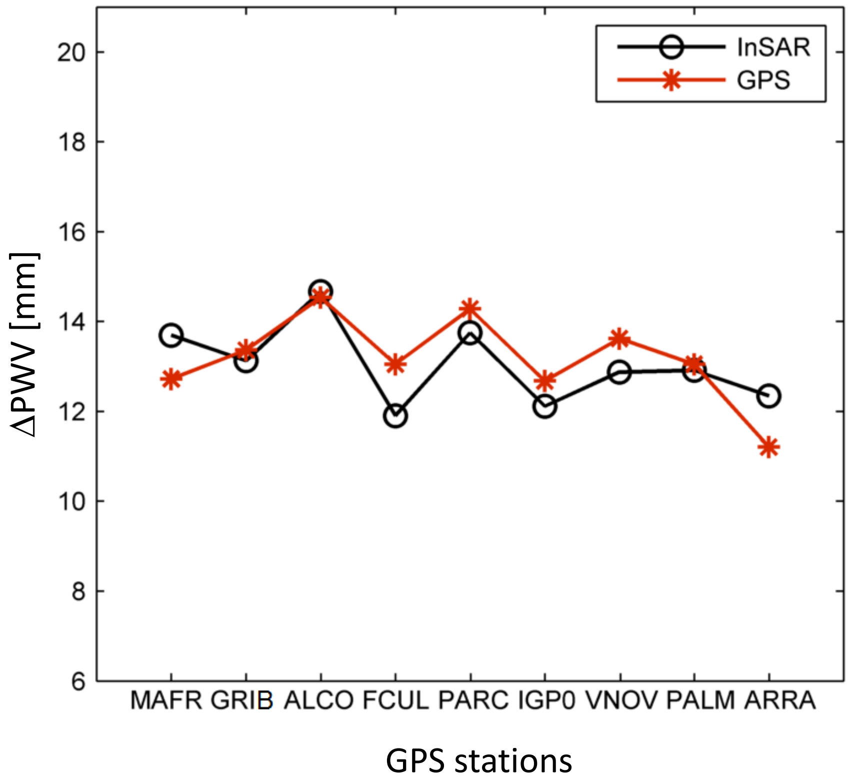




Precipitable Water Vapour and InSAR

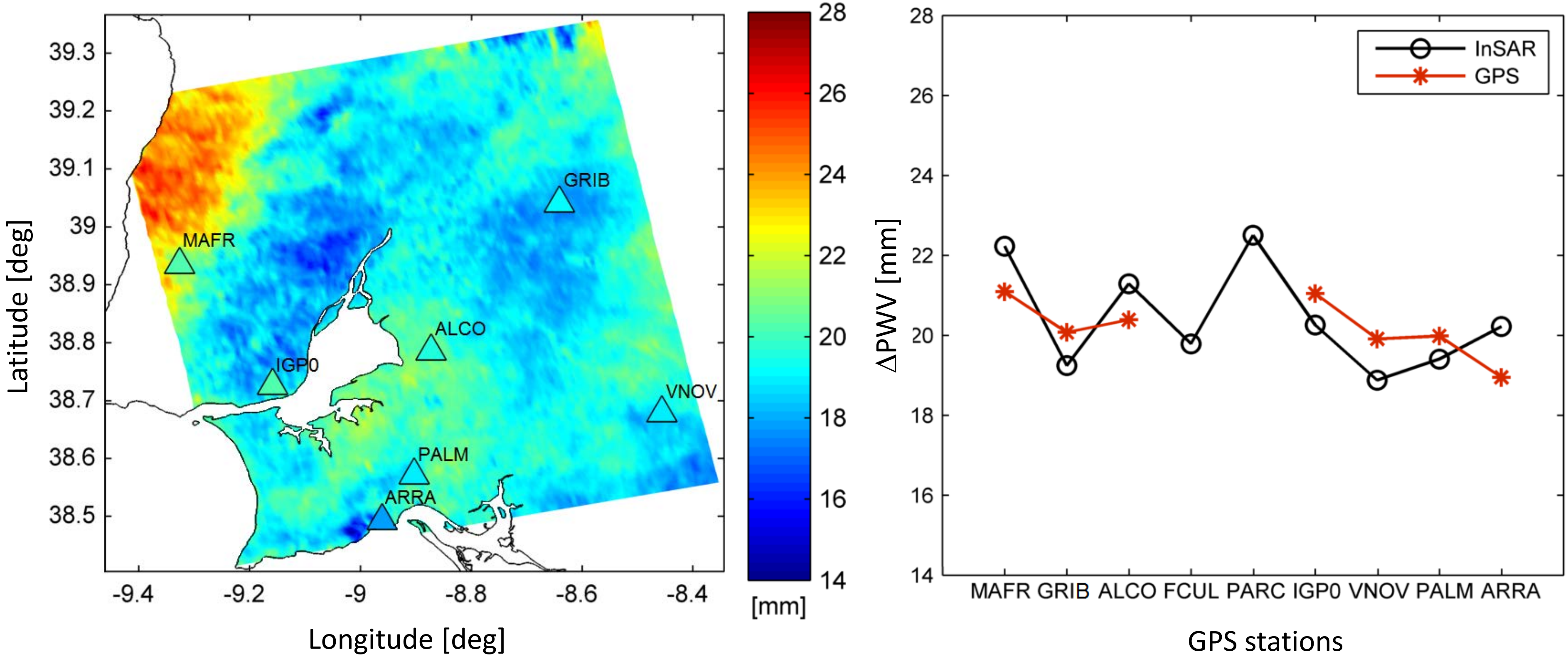
21/06/2009 → 26/07/2009





Precipitable Water Vapour and InSAR

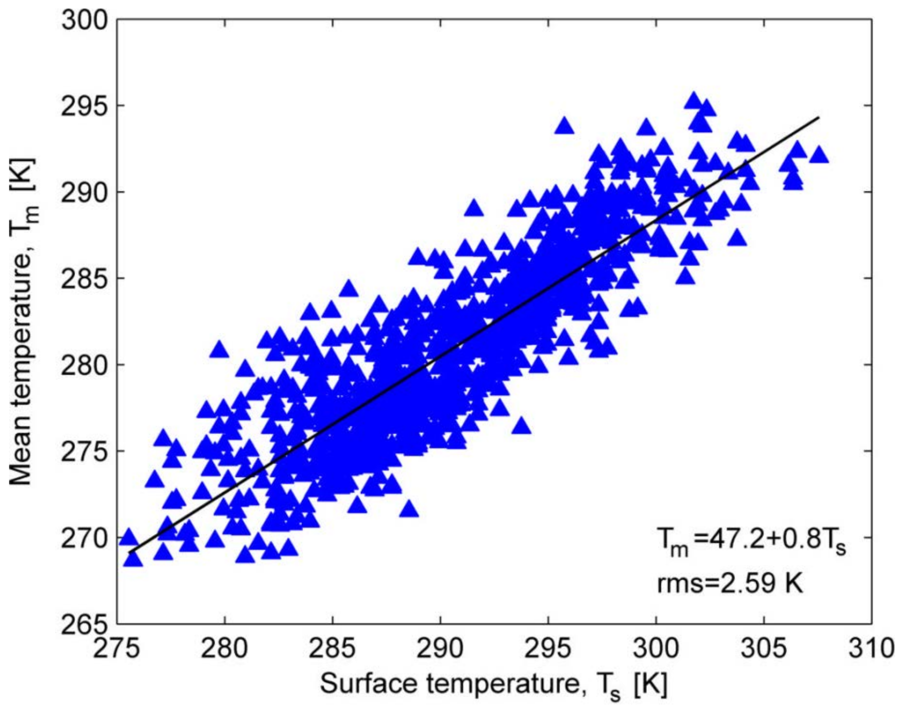
26/07/2009 → 30/08/2009



Precipitable Water Vapour and InSAR

A refinement of the ΔPWV can be obtained by accurately estimating the mean vertical temperature used to compute the constant Π.

Usually Tm is obtained by a linear regression with the surface temperature Ts

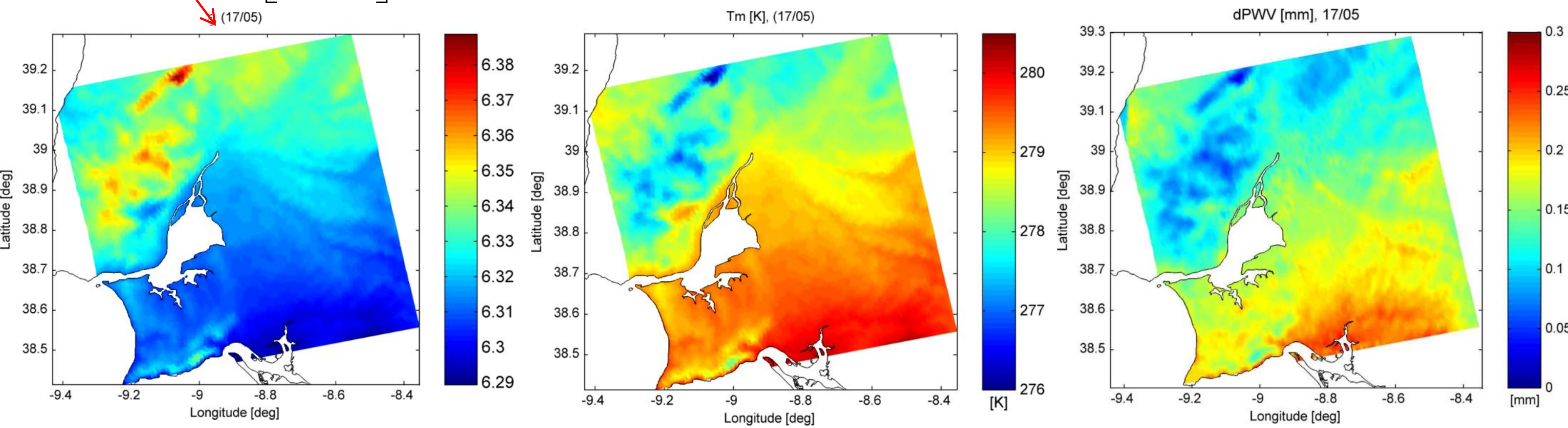


Precipitable Water Vapour and InSAR

A refinement of the ΔPWV can be obtained by properly estimating the mean vertical temperature used to compute the constant Π.

Usually Tm is obtained by a linear regression with the surface temperature Ts

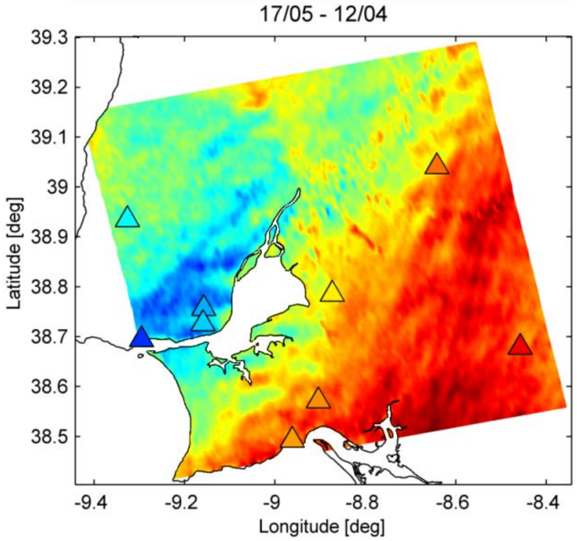
$$\xi = 10^{-6} R_v \cdot \left[\frac{k_3}{T_m} + k_2^{'} \right]$$



Mateus et al., "Maps of PWV Temporal Changes by SAR Interferometry: A Study on the Properties of Atmosphere's Temperature Profiles". IEEE Geoscience and Remote Sensing Letters, 11(12), 2065-2069, 2014.

How to get estimates of absolute PWV?

Relative PWV
InSAR
17/05/09 – 12/04/09



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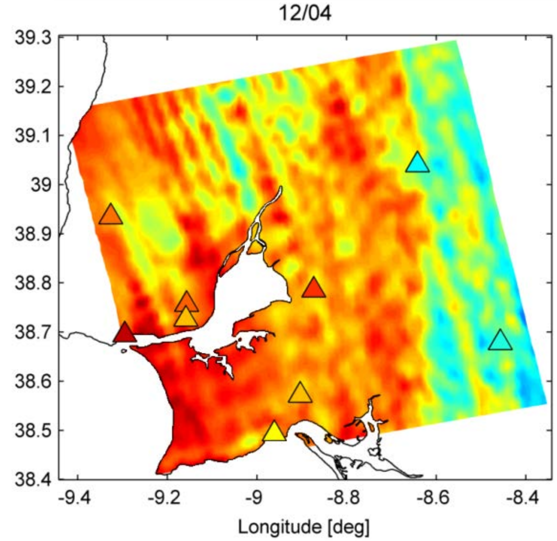
Absolute PWV
WRF model
12/04/09



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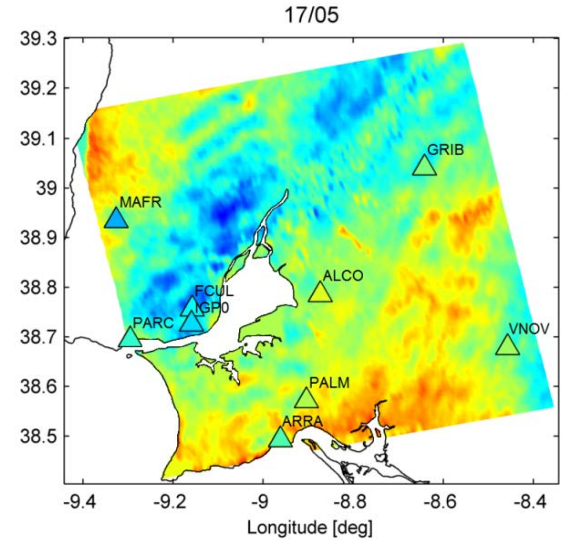
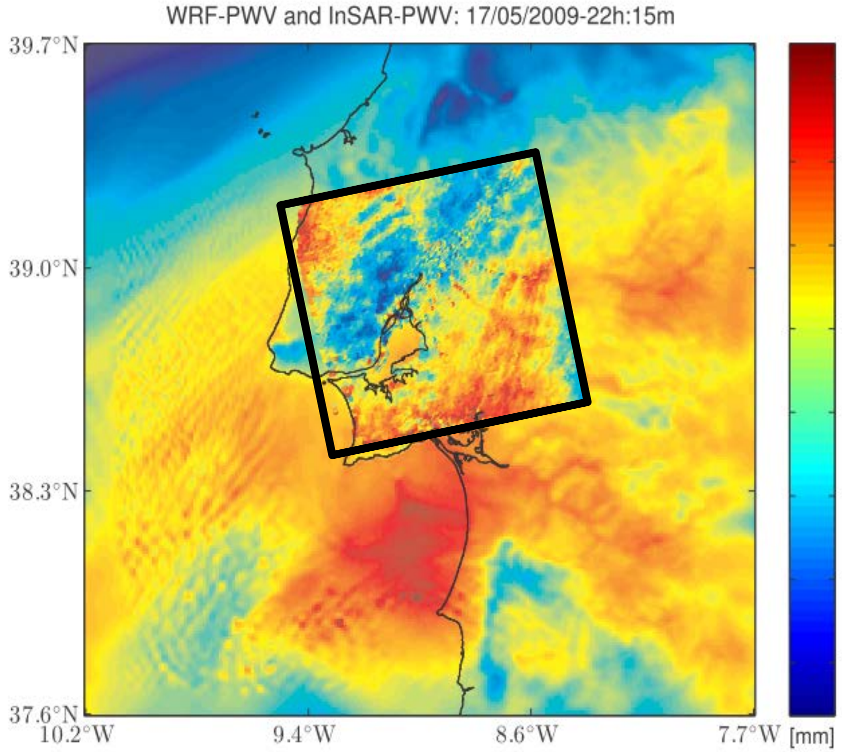
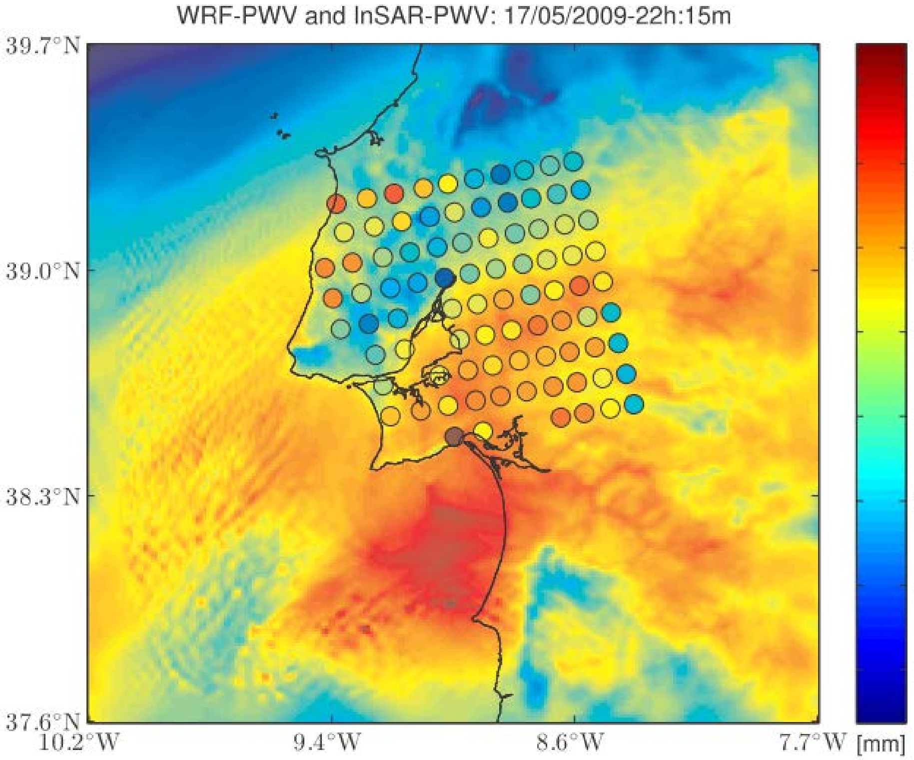
Absolute PWV
17/05/09



What can we do with InSAR estimates of PWV?

InSAR interpolation to 10km x 10km grid





Short temporal baseline approach

Given N+1 SAR images, sorted by acquisition times, we generate N interferograms by selecting the (master, slave) couples with the shortest temporal baseline.

We denote with $t_S = (t_{S1}, \ldots, t_{SN})$ and $t_M = (t_{M1}, \ldots, t_{MN})$ the acquisition times of master and slave images used to generate the N interferograms ($t_{mj} > t_{sj}$)

Considering the j-th interferogram , in every pixel we have

$$\varphi_j = \frac{4\pi}{\lambda} \cdot \frac{1}{\Pi \cdot M(\vartheta)} \left[PWV\left(0, t_{Sj}\right) - PWV\left(0, t_{Mj}\right) \right] = \psi(t_{Sj}) - \psi(t_{Mj})$$

The problem consists in estimating the unknown vector $\psi = (\psi(t_1), \ldots, \psi(t_{N+1}))$ from the vector of known interferogram phase values $\varphi = (\varphi(t_1), \ldots, \varphi(t_N))$

$$\min_{\psi \in \Re^{N+1}} \left\| A\psi - \varphi \right\|^2$$

Short temporal baseline approach

Supposing that $\psi(t)$ can be expressed as a linear model depending on a vector p of lenght L, the problem can re-stated as finding the unknown vector p from measured values $\varphi = (\varphi(t_1), \ldots, \varphi(t_N))$

$$\min_{\psi \in \Re^{N+1}} \left\| Sp - \varphi \right\|^2$$

The structure of matrix S depends on how S is related to $\psi(t)$

If $\psi(t)$ has dimensionality L, the final problem is given by

$$\min_{\psi \in \Re^{L}} \left\| \tilde{S}\,\tilde{p} - \varphi \right\|^2$$

Cubic B-spline model

Definition 3.1 *Given a sorted sequence* $\{T_\nu\}_{\nu \in \mathbb{N}} \subset \mathbb{R}$, *a* ***B-spline*** *of degree* n *is a spline function* $s(t)$ *such that*

- $s(t)$ *is non-zero only within an interval* $[T_\nu, T_{\nu+n+1}]$ *for a certain* ν
- $\int_{T_\nu}^{T_{\nu+n+1}} s(t)dt = 1$
- *in any interval* $[T_i, T_{i+1}]$ $i = \nu, \ldots, \nu+n$ *is a polynomial of degree at most* n
- $s(t) \in C^{(n-1)}(\mathbb{R})$

Such function $s(t)$ *is usually noted as* $B_{n,\nu}(t)$.

For example a cubic B-spline (n=3) has a support $[T_\nu, T_{\nu+n+1}]$ and is a cubic polinomial in in any of the four intervals $[T_\nu, T_{\nu+1}]$, $[T_{\nu+1}, T_{\nu+2}]$, $[T_{\nu+2}, T_{\nu+3}]$, $[T_{\nu+3}, T_{\nu+4}]$ composing its support

Cubic B-spline model

If a n-degree spline function s(t) is defined on an interval A partitioned by a finite sequence $t_0 < t_1 < \dots t_{M+1}$, it can be expressed as

$$s(t) = \sum_{\nu=-n}^{m} p_{\nu} \cdot B_{n,\nu}(t)$$

where $t_{-N} < \dots t_{-1} < t_0 < t_1 < \dots t_{M+1} < \dots t_{M+N+1} = t_L$ are called <u>control points</u>

Thanks to this theorem the approximation by means of n-th degree splines can be achieved by a sum of B-splines provided each interval $[t_i, t_j]$ contains at least an observation point the Schoenberg-Whitney condition

↓

Useful for studying ΔPWV(t) characterized by
a good spatial correlation but poor temporal correlation

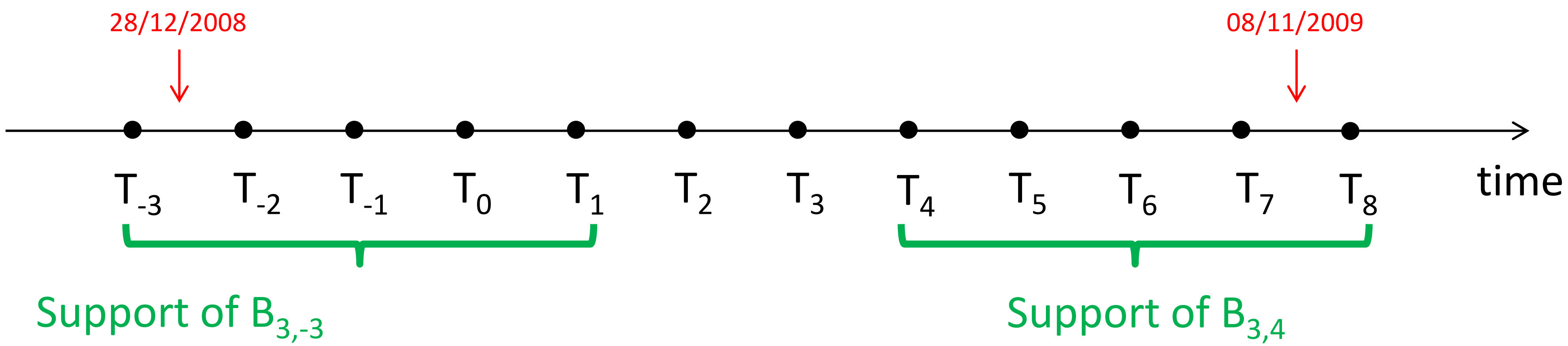
Experiment

A set of 11 Envisat-ASAR images acquired over the Lisbon area are used

N.	Master acquisition date	Slave acquisition date
1	28-12-2008	23-11-2008
2	28-12-2008	01-02-2009
3	08-03-2009	01-02-2009
4	08-03-2009	12-04-2009
5	17-05-2009	12-04-2009
6	17-05-2009	21-06-2009
7	26-07-2009	21-06-2009
8	26-07-2009	30-08-2009
9	04-10-2009	30-08-2009
10	04-10-2009	08-11-2009

$n = 3$

$m = L - (n+1) = L-4$



Cubic B-spline model

In our case we can use B-splines so that we express

$$PWV(t) = \sum_{\nu=-3}^{L-4} p_\nu \cdot B_{3,\nu}(t)$$

The j-th interferogram is then given by

$$\varphi_j = \psi(j) - \psi(j-1) = \frac{4\pi}{\lambda} \cdot \frac{1}{\Pi \cdot M(\vartheta)} \sum_{\nu=-3}^{L-4} p_\nu \cdot \left[B_{3,\nu}(j) - B_{3,\nu}(j-1) \right]$$

and we can formulate the linear system

$$\mathbf{S} \cdot \mathbf{p} = \varphi$$

where S is generally a sparse matrix. Then the problem to be solve is

$$\min_{\psi \in \Re^L} \left\| \tilde{\mathbf{S}} \tilde{\mathbf{p}} - \varphi \right\|^2$$

Spatial block system and physical constraints

We adopt the B-splines model with a number L = 7 of B-splines. For a given pixel the least-squares problem in solved in R^7

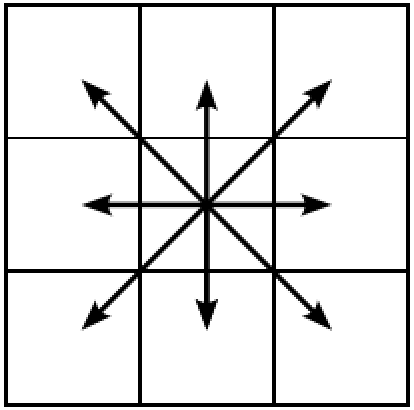
$$\min_{\psi \in \Re^L} \left\| \tilde{S}\,\tilde{p} - \varphi \right\|^2$$

The interferograms are partitioned in 3x3 non-overlapping windows and the least squares problem is jointly solves for the 9 pixels of each window. The dimensionality of the problem increases to 7x9 = 63.

$$-b_l \leq p_P(l) - p_{Q_P}(l) \leq -b_l \qquad with \quad l = 1, \ldots, L$$

Spatial block system and physical constraints

Linear constraints are added by imposing bounds on element of **p** relating adjacent pixels following the scheme (a total number of 7x8 = 56 box constraints are obtained for each problem



$$-b_l \leq p_P(l) - p_{Q_P}(l) \leq -b_l \qquad with \quad l = 1, \ldots, L$$

Spatial block system and physical constraints

$$\min_{\Re^{63}} \left\| \begin{pmatrix} S & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \\ 0 & & & S \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ \vdots \\ p_9 \end{pmatrix} - \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_9 \end{pmatrix} \right\|^2$$

subject to (b>0)

$$-b \leq p_P(1) - p_{Q_P}(1) \leq b$$

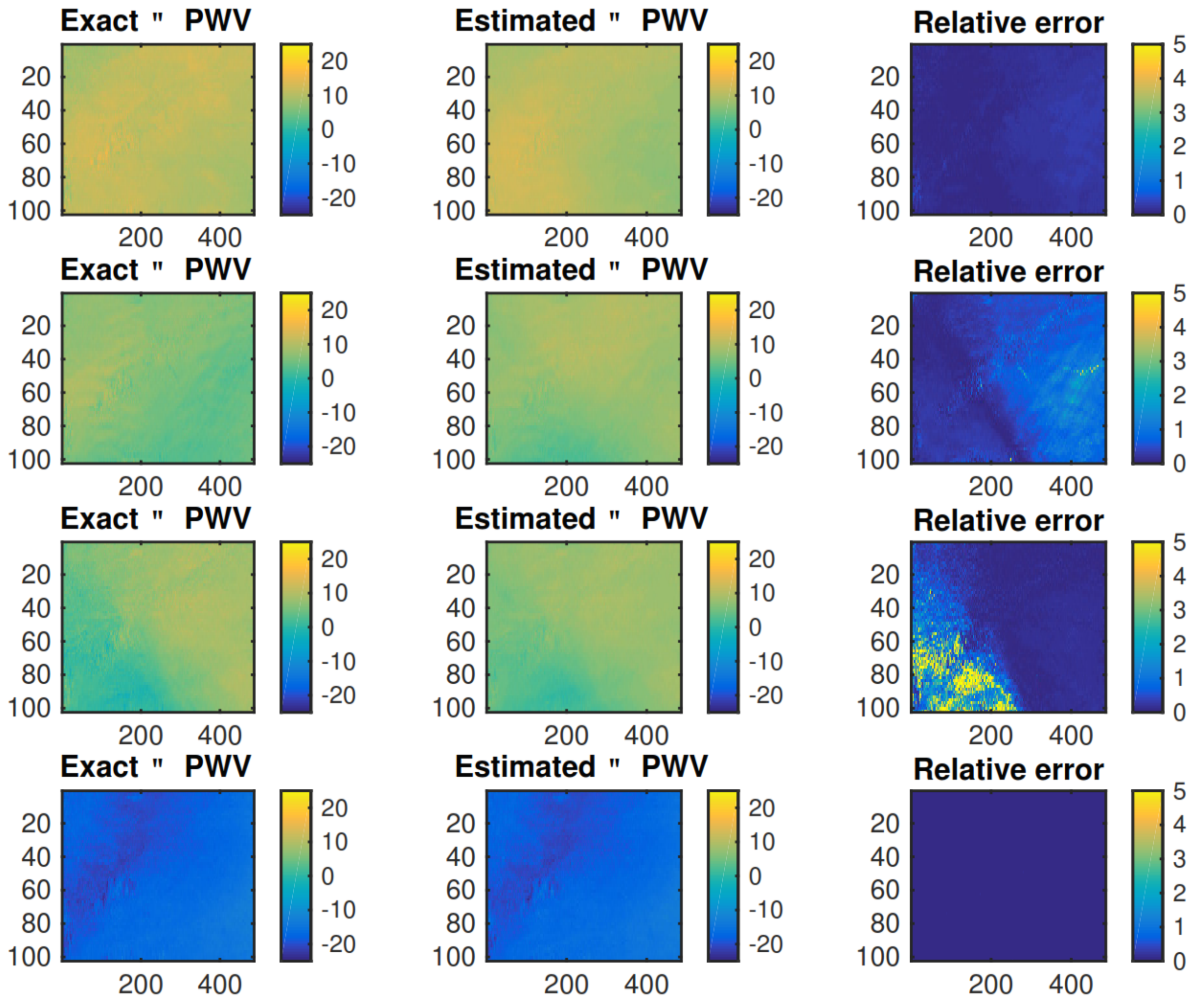
$$\vdots$$

$$\underbrace{-b \leq p_P(7) - p_{Q_P}(7) \leq b}_{\text{For each pair of adjacent pixels}}$$

The same value of b>0 is used for all constraints

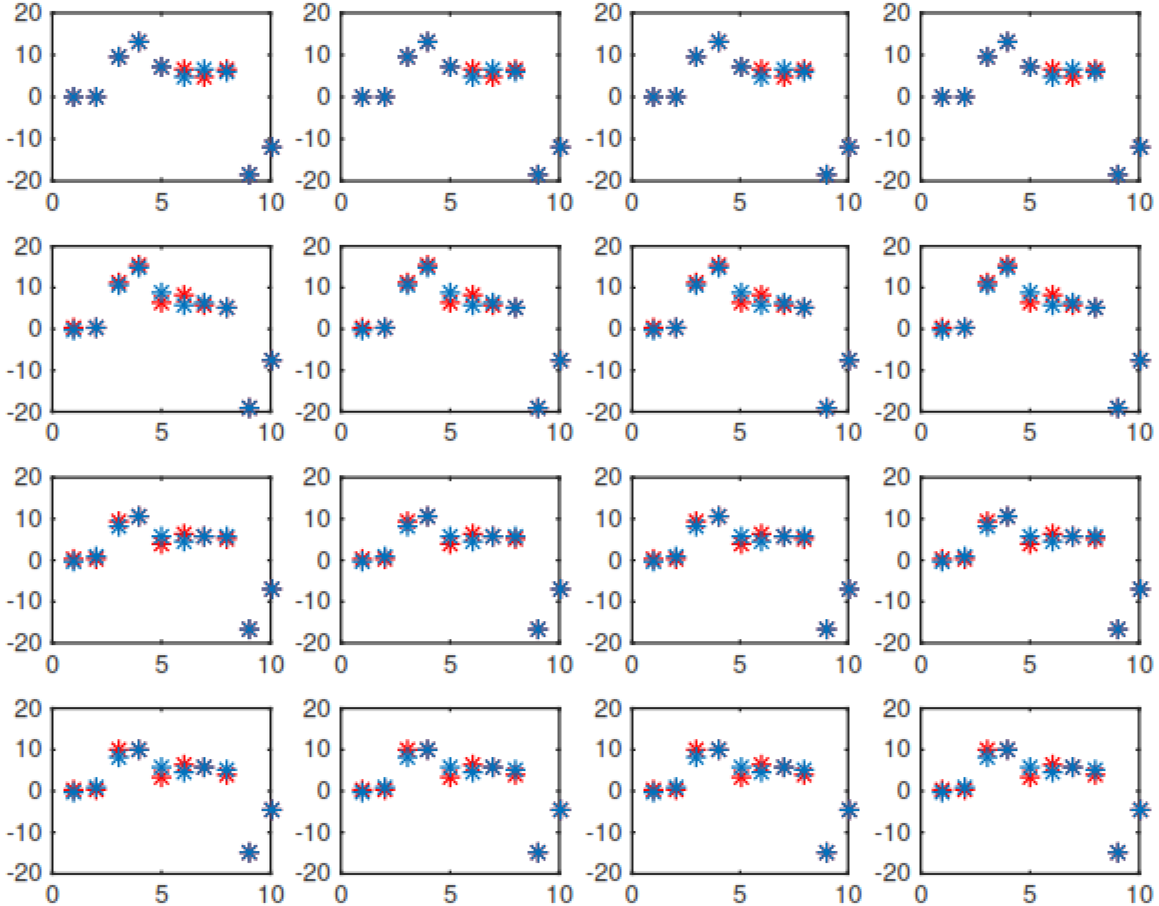
<u>The aim is to find a good continuous-time model for PWV(t)</u>

Results (b = 20)

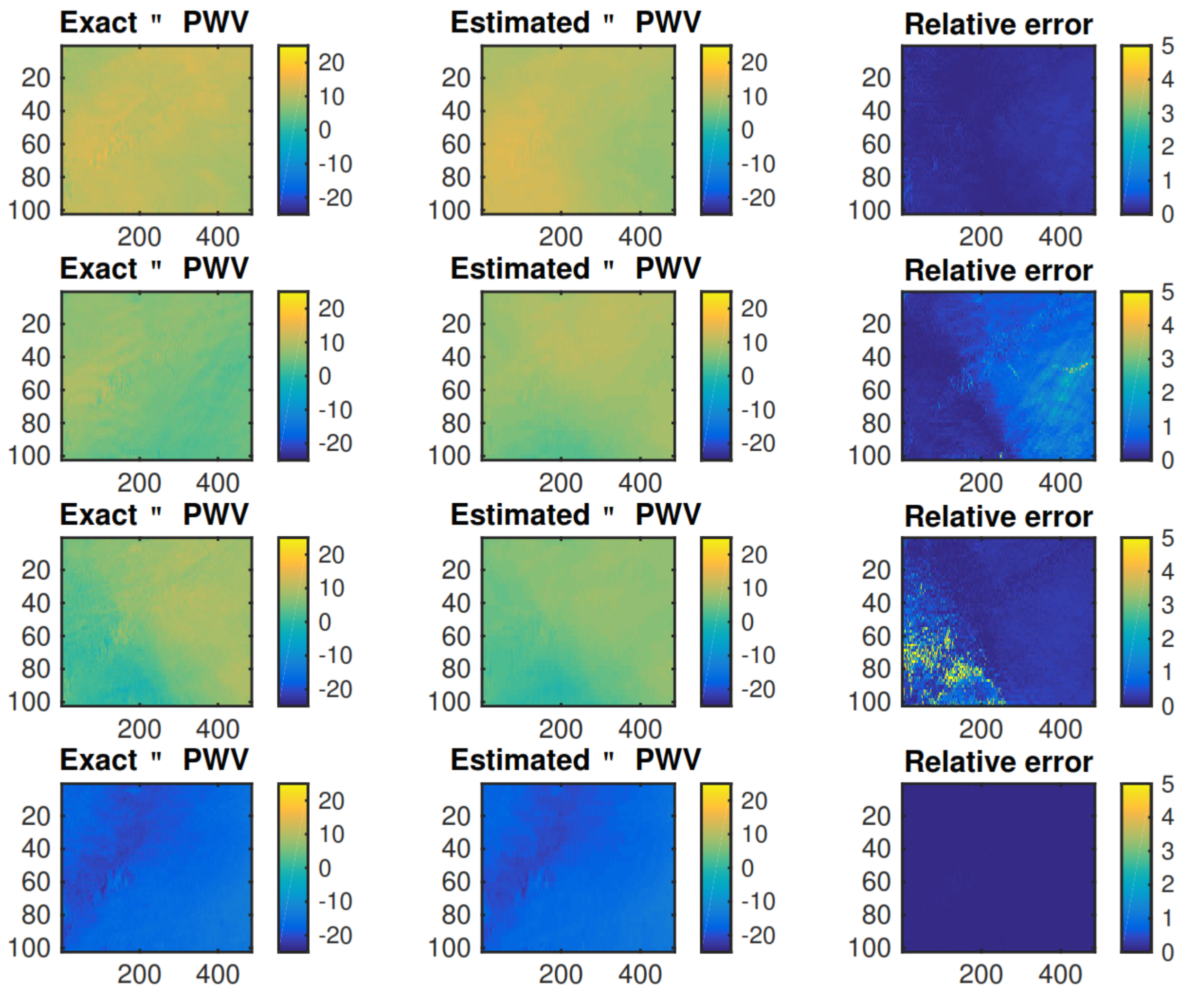


Exact " PWV
Estimated " PWV
Relative error
Exact " PWV
Estimated " PWV
Relative error
Exact " PWV
Estimated " PWV
Relative error
Exact " PWV
Estimated " PWV
Relative error

Results (b = 20)

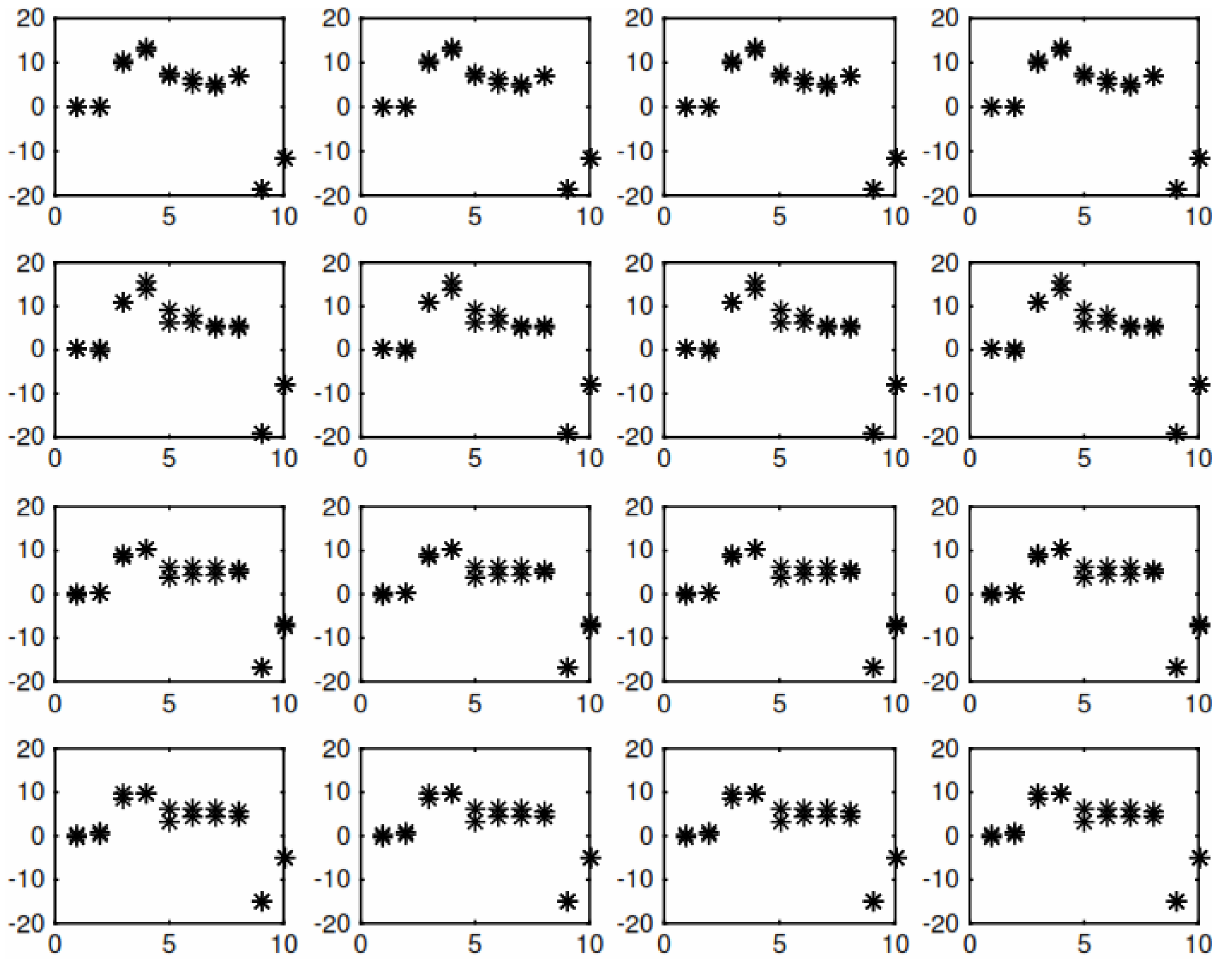


Results (b = 0.1)



Exact " PWV
Estimated " PWV
Relative error
Exact " PWV
Estimated " PWV
Relative error
Exact " PWV
Estimated " PWV
Relative error
Exact " PWV
Estimated " PWV
Relative error

Results (b = 0.1)



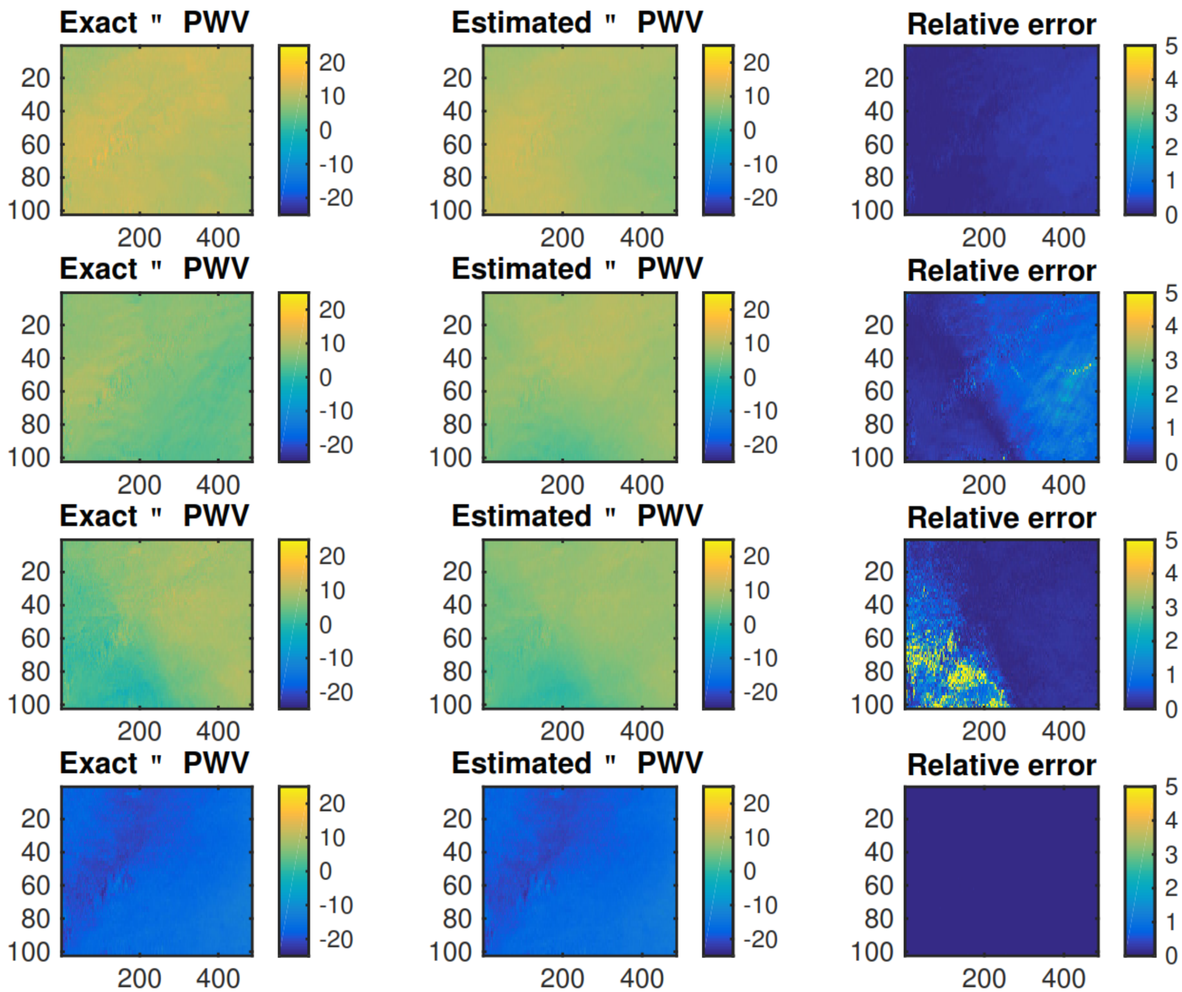
Conclusions and future work

• The original SBAS method has been adopted to generate maps of PWV and extended by adding spatial constraints to help in finding a suitable solution

•The problem modelling temporal evolution of PWV maps generated by InSAR has been studied by means of B-splines

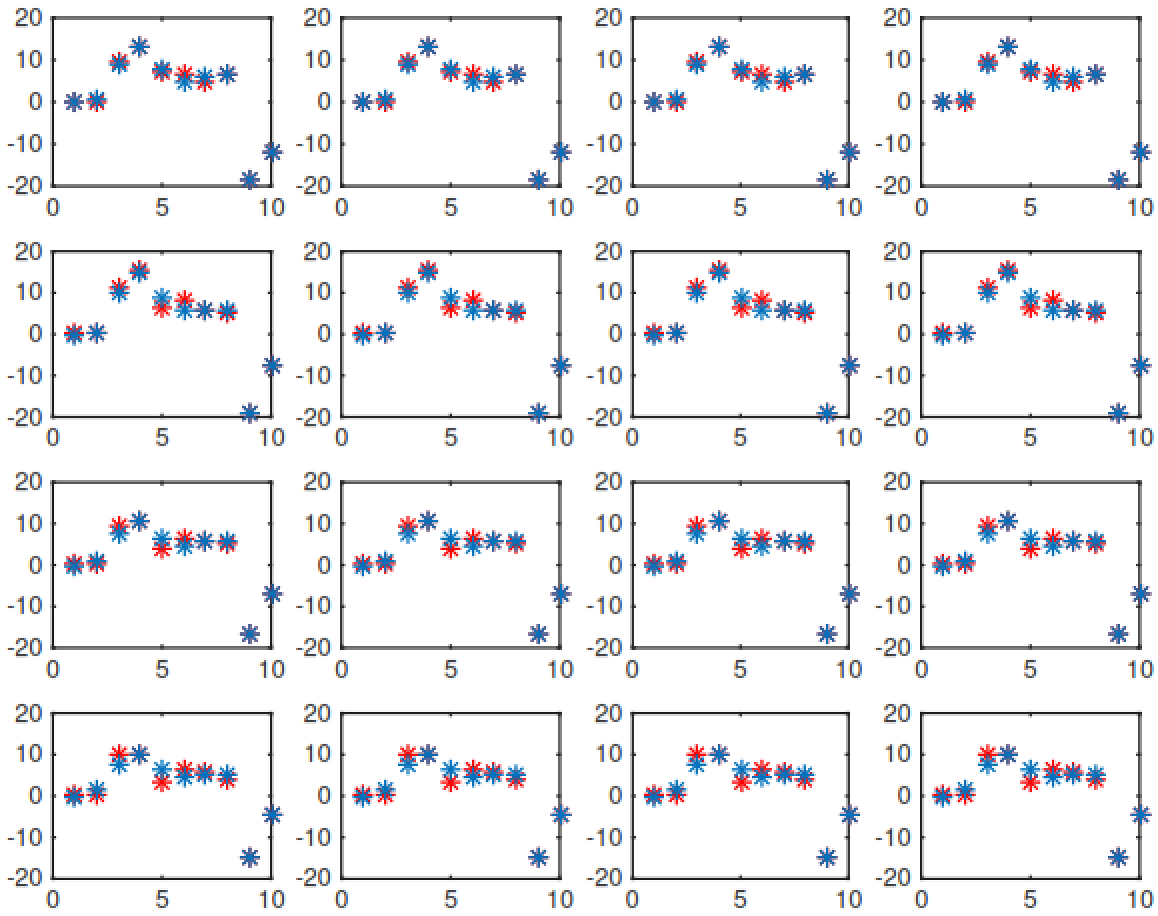
• The future work will focus on extending the proposed method to include GPS-derived PWV time series

Results (b = 10)

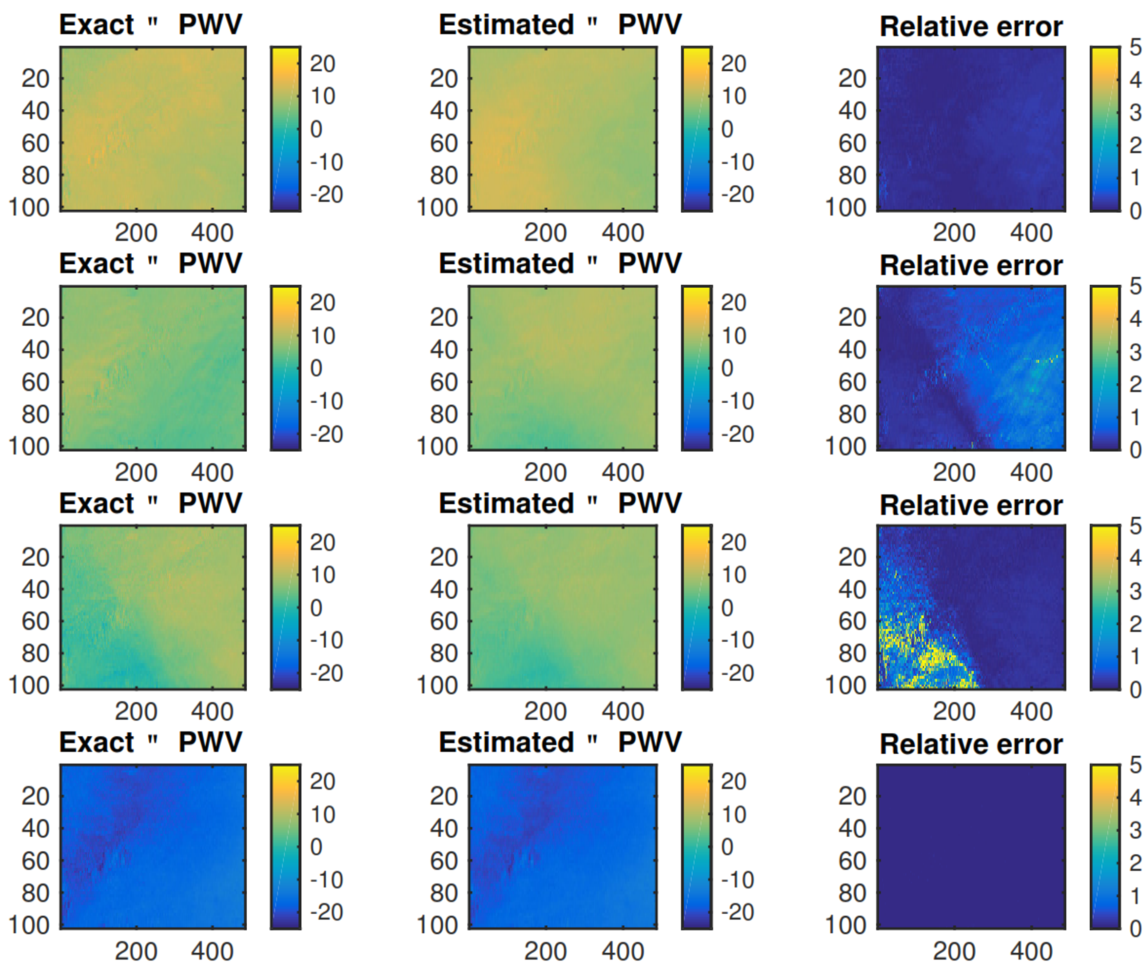


Exact " PWV
Estimated " PWV
Relative error
20
40
60
80
100
200 400
20 10 0 -10 -20
5 4 3 2 1 0

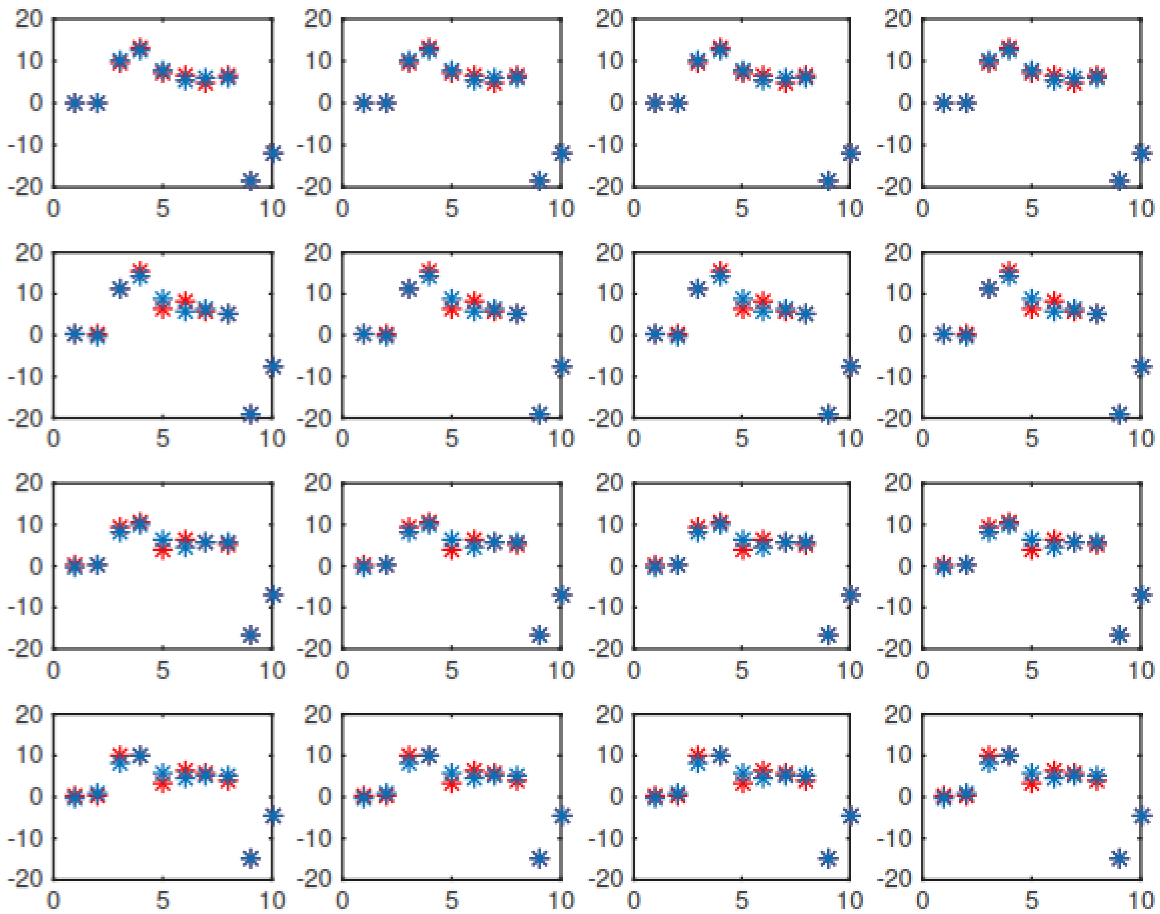
Results (b =10)



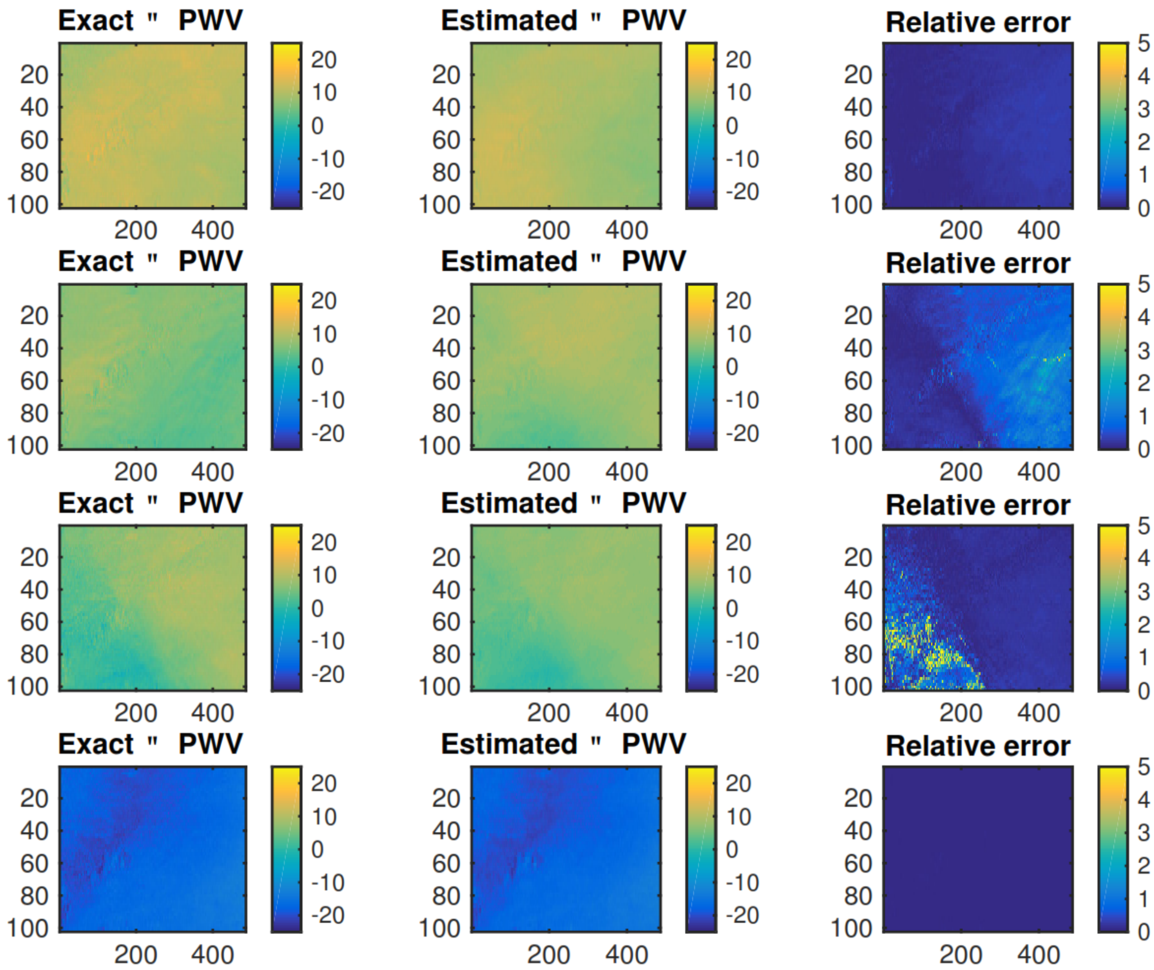
Results (b = 5)


Exact " PWV
Estimated " PWV
Relative error
20
40
60
80
100
200
400
20
10
0
-10
-20
5
4
3
2
1
0

Results (b =5)

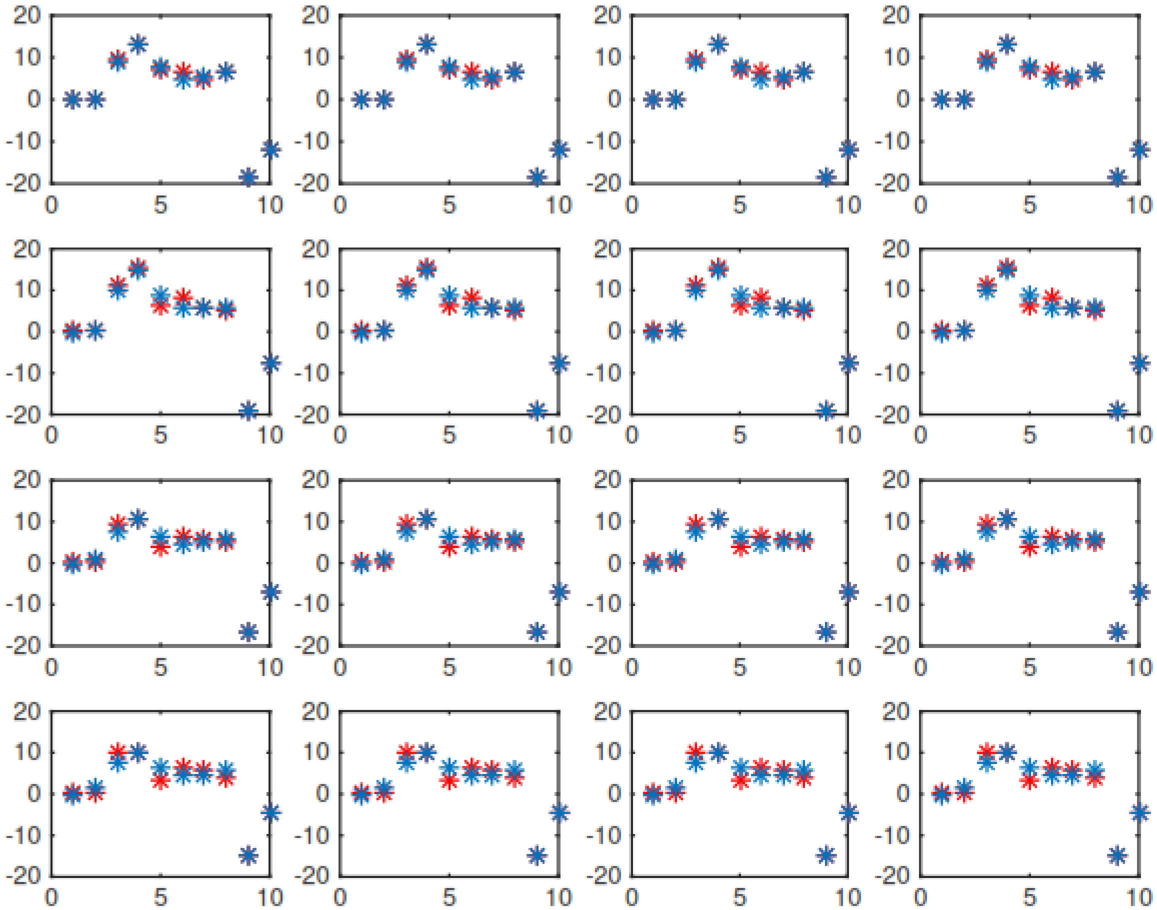


Results (b = 3)

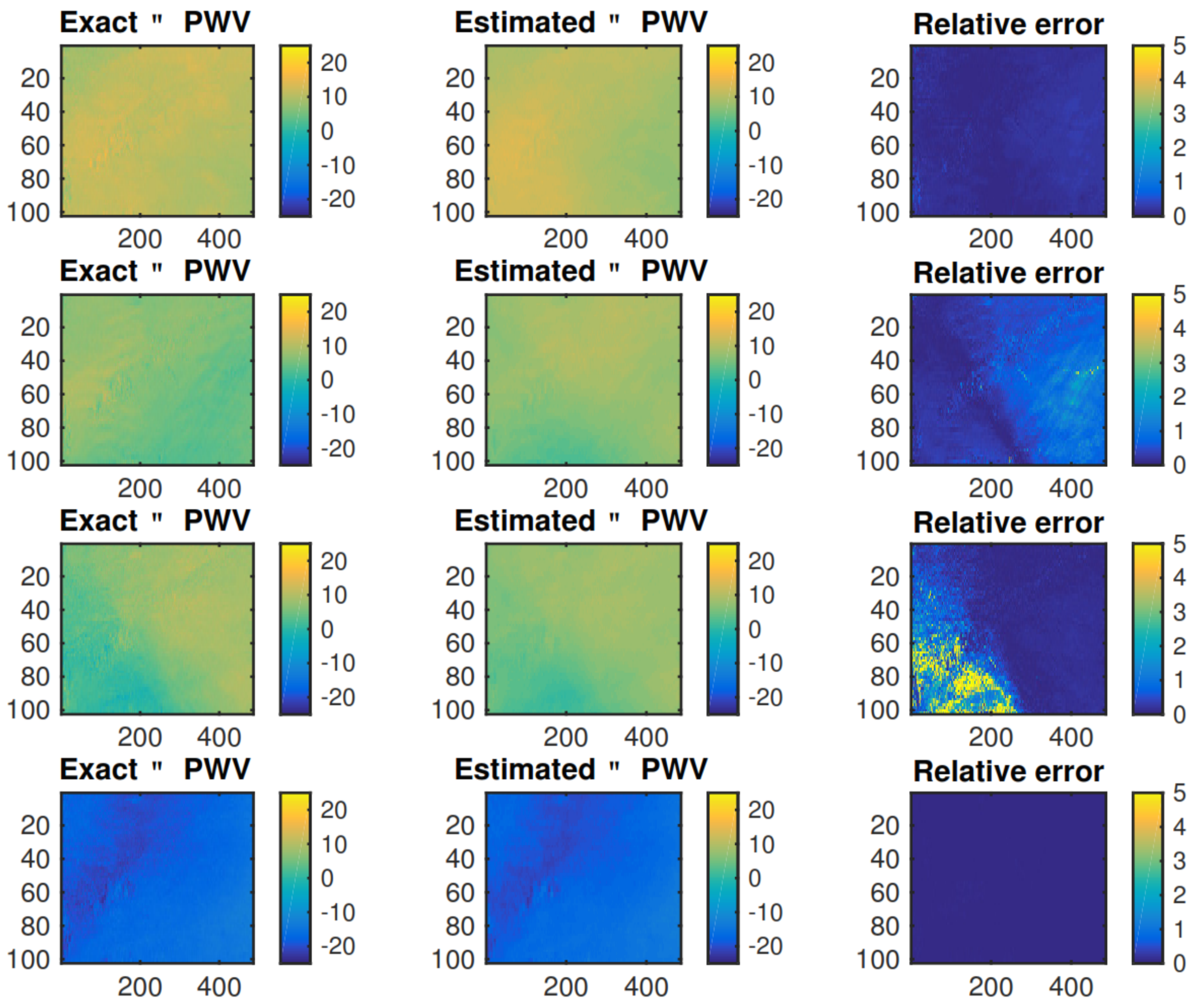


Exact " PWV
Estimated " PWV
Relative error
20
40
60
80
100
200
400
20
10
0
-10
-20
5
4
3
2
1
0

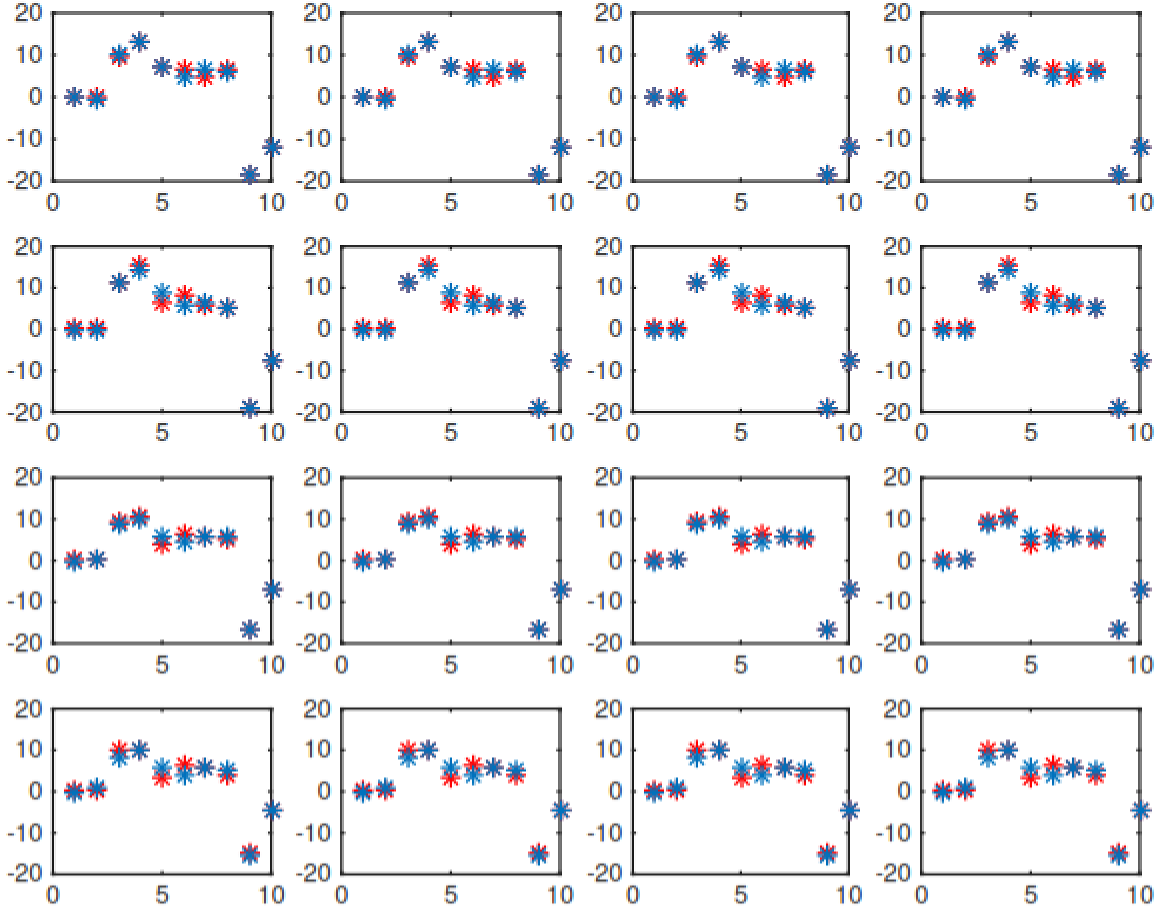
Results (b = 3)



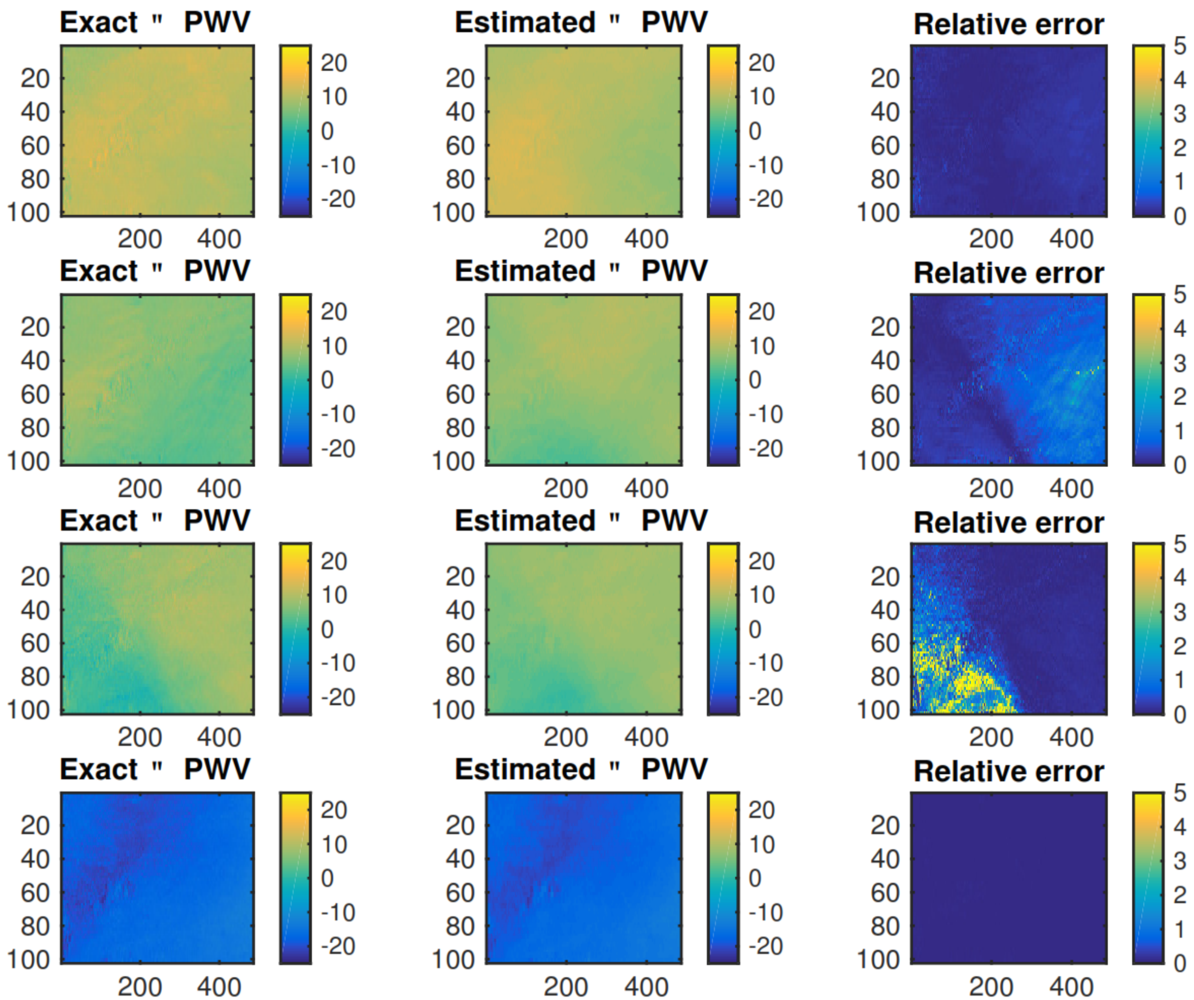
Results (b = 1)


Exact " PWV
Estimated " PWV
Relative error
20
40
60
80
100
200
400
20
10
0
-10
-20
5
4
3
2
1
0

Results (b = 1)



Results (b = 1)


Exact " PWV
Estimated " PWV
Relative error
20
40
60
80
100
200
400
20
10
0
-10
-20
5
4
3
2
1
0

Results (b = 1)